Student #:
Name:

Write down answers in-between questions. Please answer using short sentences.

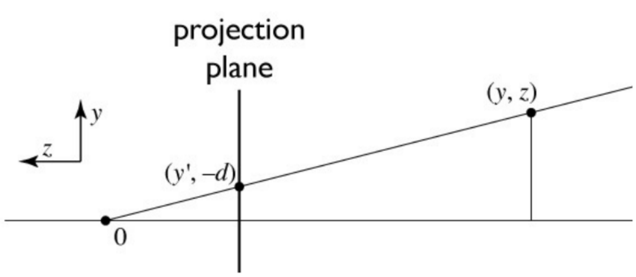
1. How many bytes are necessary to store a 1024×1024 color image without an alpha channel using 8 bits per channel?

2. Fill in the blanks below in the source code for the incremental linear interpolation algorithm that calculates qRow=cx*x + cy*y+ck for all pixels.

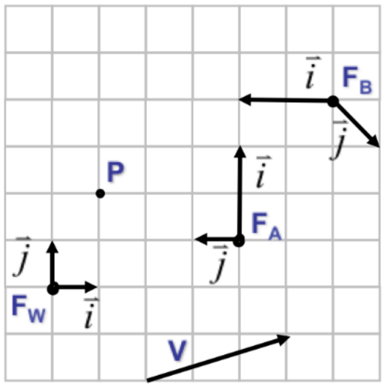
```
linEval(xl, xh, yl, yh, cx, cy, ck) {

   // setup
   qRow = cx*xl + cy*yl + ck;

   // traversal
   for y = yl to yh {
      qPix = qRow;
      for x = xl to xh {
         output(x, y, qPix);
         qPix +=□x;
      }
      qRow +=□y;
   }
}
```

$c_x = .005;\ c_y = .005;\ c_k = 0$
(image size 100x100)

3. Write down the 4×4 projection matrix that maps a 3d point (x,y,z) to (x',y',z)?
Hint: similar triangles, homogeneous coordinates



4. Coordinate Frames and Homogeneous Coordinates. Hint: transform basis vectors.



$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_W = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_B = \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_A$$

(a) Express point P in each of the three coordinate frames. F_W, F_A, F_B.

(b) Express vector V in each of the three coordinate frames.

(c) Find the 3 × 3 affine transformation matrix which takes a point from F_A and expresses it in terms of F_W . I.e., determine M, where $P_W = M\ P_A$. Write your answer in the space to the right of the diagram above.

(d) Find the 3 × 3 affine transformation matrix which takes a point from F_A and expresses it in terms of F_B . I.e., determine M, where $P_B = M\ P_A$. Write your answer in the space to the right of the diagram above.

5. What is the equation of the plane through 3D points **a**, **b**, and **c**? What is the normal vector to this plane? You can use any vector operators in the answer.

6. Given a rigid transformation matrix R of which the linear part Q is an orthonormal matrix, derive the inverse of the matrix R.

$$R = \begin{bmatrix} Q & \mathbf{u} \\ 0 & 1 \end{bmatrix}, \quad R^{-1} =$$

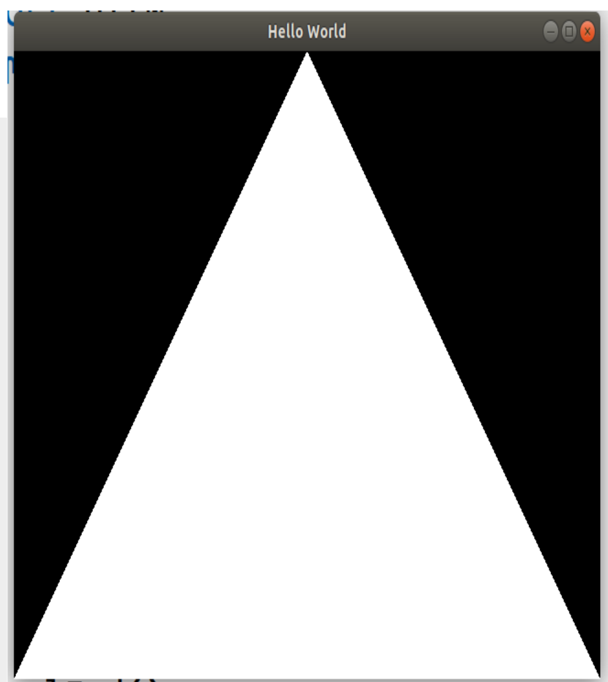
7. (a) Write down the 4 × 4 3D matrix to move by (x_m, y_m, z_m).

(b) Write down the 4 × 4 3D matrix to rotate by an angle θ about the y-axis.

(c) Write down the 4 × 4 rotation matrix M that maps the orthonormal 3D vectors $u = (x_u, y_u, z_u)$, $v = (x_v, y_v, z_v)$, and $w = (x_w, y_w, z_w)$, to orthonormal 3D vectors $a = (x_a, y_a, z_a)$, $b = (x_b, y_b, z_b)$, and $c = (x_c, y_c, z_c)$, so that $Mu = a$, $Mv = b$, and $Mw = c$.

8. Shown on the right is the result using the render function shown below. What would be drawn if the commented lines in the render function are uncommented. Overdraw the result on the captured screen below.

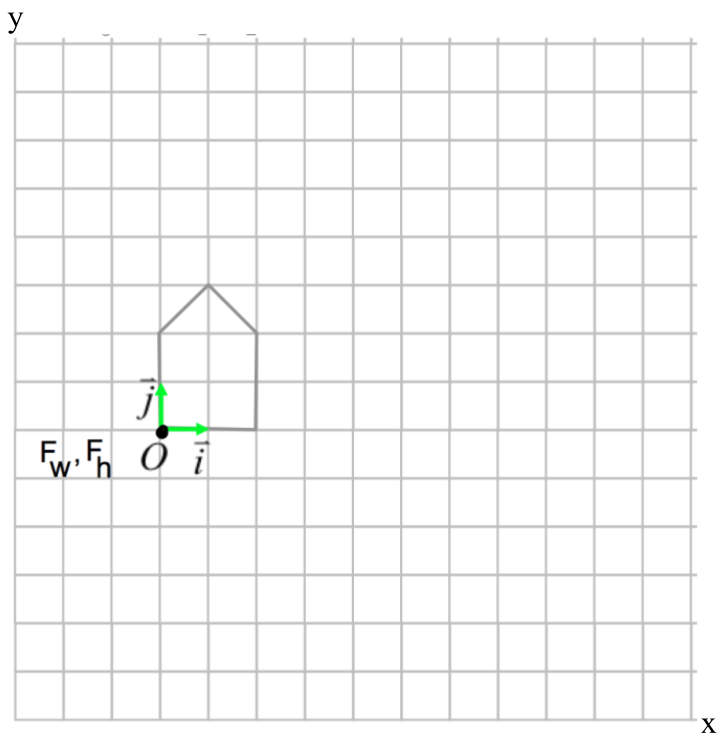
```
def render():
    glClear(GL_COLOR_BUFFER_BIT)
    glLoadIdentity()
    #glScalef(0.5,1,1)
    #glTranslatef(1,0,0)
    glBegin(GL_TRIANGLES)
    glVertex2f(0.0, 1.0)
    glVertex2f(-1.0,-1.0)
    glVertex2f(1.0,-1.0)
    glEnd()
```



9. Consider the house which is shown below in its untransformed state, e.g., $F_h = F_{World}$.

$$M_1 = Trans(0,5,0)Rot(z,-90)Trans(-1,3,0),$$
$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$M_3 = M_1 M_2$$

(a) Sketch the intermediate and final transformations of the house for the transformations that make up M_1 . Apply matrices from left to right in sequence. Draw the final coordinate frame as well, and label it with F_1.

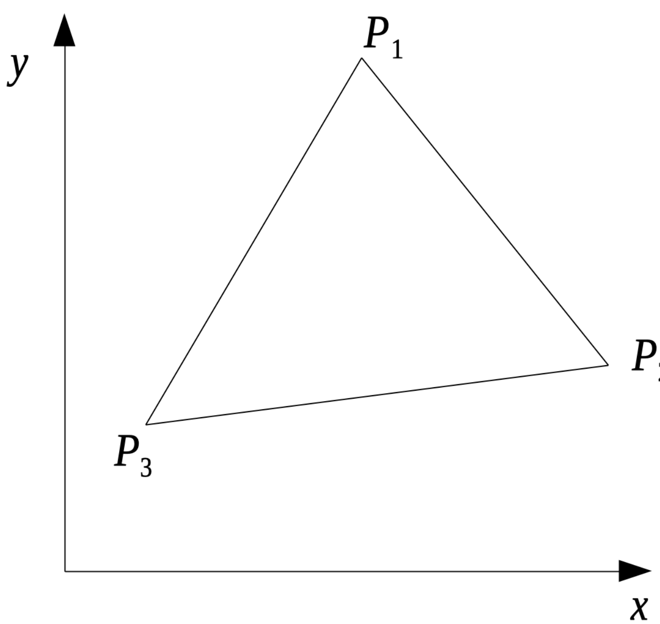


(b) Sketch the coordinate frame and house that would result from applying transformation matrix M_3 to the house object, i.e., apply the M_2 transformation to your result in part (a). Label it with F_3.

10. A linear function on a plane is $q(x, y) = c_x x + c_y y + c_k$. To interpolate parameters across a triangle we need to find the c_x, c_y, and c_k that define the (unique) linear function that matches the given values q_0, q_1, q_2 at all 3 vertices (x_0, y_0), (x_1, y_1), (x_2, y_2). Derive c_x, c_y, and c_k using the given values. You can use a matrix inversion in the answer.

11. Barycentric Coordinates

Assume that the barycentric coordinates are defined according to $P = \alpha P_1 + \beta P_2 + \gamma P_3$.



(a) On the diagram above, label the vertices with their barycentric coordinates, (α, β, γ).Then sketch the three lines that correspond to $\gamma = 0, \gamma = 0.5, \gamma = 1$.

(b) Sketch the point P that corresponds to $\alpha = 0.5, \beta = 0.3$.

12. (a) Given two nonparallel, three-dimensional vectors u and v, how can we form an orthogonal coordinate system in which u is one of the basis vectors? Calculate all the basis vectors using the cross product operator (×) and the length operator (| |).

(b) Why is it important that the vectors u,v from (a) be non-parallel?

13. The official projection matrix is given by $\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

(a) Verify that this matrix scales down all the positions on the far plane.

(b) Explain what the scale factor is, and why the scale factor is designed in such a way.