Student #:
Name:

Write down answers in-between questions. Please answer using short sentences.

1. How many bytes are necessary to store a 1024×1024 color image without an alpha channel using 8 bits per channel ?

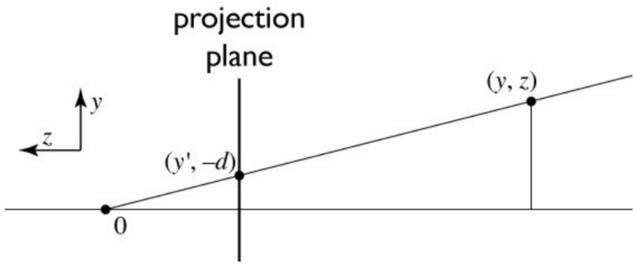
2. Show by counterexample that is is not always true that for 3D vectors a,b, and c, $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = (\boldsymbol{a} \times \boldsymbol{b}) \times \boldsymbol{c}$.

3. Fill in the blanks below in the source code for the incremental linear interpolation algorithm that calculates qRow=cx*x + cy*y+ck for all pixels.

```
linEval(xl, xh, yl, yh, cx, cy, ck) {

   // setup
   qRow = cx*xl + cy*yl + ck;

   // traversal
   for y = yl to yh {
      qPix = qRow;
      for x = xl to xh {
         output(x, y, qPix);
         qPix +=
      }
      qRow +=
   }
}
```

$c_x = .005;\ c_y = .005;\ c_k = 0$
(image size 100x100)

4. Write down the 3×4 projection matrix that maps a 3d point (x,y,z) to (x',y')?
Hint: similar triangles, homogeneous coordinates



5. What is the equation of the plane through 3D points **a**, **b**, and **c**? What is the normal vector to this plane? You can use any vector operators in the answer.

6. Given a rigid transformation matrix R of which the linear part Q is an orthonormal matrix, verify that the inverse of orthormal matrix is as below.

$$R = \begin{bmatrix} Q & \mathbf{u} \\ 0 & 1 \end{bmatrix}, \quad R^{-1} = \begin{bmatrix} Q^T & -Q^T\mathbf{u} \\ 0 & 1 \end{bmatrix}$$

7. What are the ray parameters of the intersection points between ray $\boldsymbol{a}+\boldsymbol{b}t$ and a sphere centered at $(0,0,0)$ with radius 2?

8. Calculate the ray parameter of the intersection points between ray $(-1,-1,1)+t(-1,-1,-1)$ and an arbitrary plane $ax+by+cz+d=0$ (in terms of a, b, c, and d).

9. (a) Write down the 4×4 3D matrix to move by (x_m, y_m, z_m).

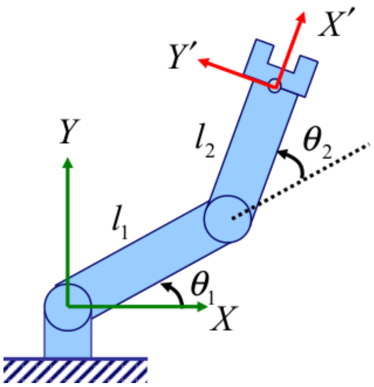
(b) Write down the 4×4 3D matrix to rotate by an angle θ about the z-axis.

(c) Write down the 4×4 rotation matrix M that maps the orthonormal 3D vectors $\boldsymbol{u}=(x_u, y_u, z_u), \boldsymbol{v}=(x_v, y_v, z_v)$, and $\boldsymbol{w}=(x_w, y_w, z_w)$, to orthonormal 3D vectors $\boldsymbol{a}=(x_a, y_a, z_a), \boldsymbol{b}=(x_b, y_b, z_b)$, and $\boldsymbol{c}=(x_c, y_c, z_c)$, so $M\boldsymbol{u}=\boldsymbol{a}, M\boldsymbol{v}=\boldsymbol{b}$, and $M\boldsymbol{w}=\boldsymbol{c}$.

10. Explain in words and using pictures what this 2D transformation matrix does:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

11. Write down the transformation matrix T of the tool (at X', Y') in terms of 2D rotation matrix R_θ and 2D translation matrix $T_{(tx,ty)}$.



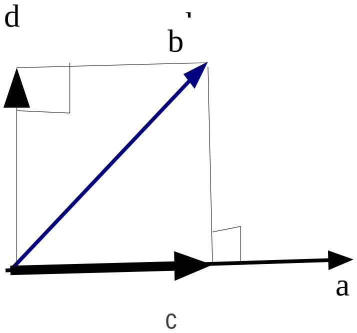
12. Shown on the right is the result using the MyDisplay function shown below. What would be drawn if the commented block in the MyDisplay function is uncommented. Overdraw the result on the captured screen below.
(a)

```
void MyDisplay()
{
    glClear(GL_COLOR_BUFFER_BIT|GL_DEPTH_BUFFER_BIT);
    glLoadIdentity();
    double width=0.3;

    glRotatef(-45, 0,0,1);
    drawBox( width);

    /*  what is drawn if this block is uncommented?
     *  Hint: glTranslatef modifies the current modelview matrix M as M*T
     *  (whenT is the translation matrix.)
    glTranslatef(2*width,0,0);
    drawBox(width);
    */

    glFlush();
}
```

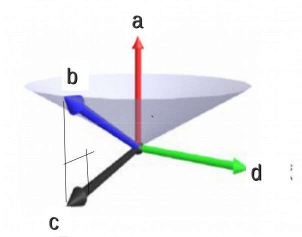
(b) Fill in the blanks below.

```
void drawBox(float height){
     glBegin(GL_POLYGON);
     glColor3f(0.5, 0.5, 0.5);
     glVertex3f(0, height,0);
     glVertex3f(0, 0, 0);
     glVertex3f(_____, _____, _____);
     glVertex3f(_____, _____, _____);
     glEnd();
}
```

13. Represent vector $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of vector $\boldsymbol{a}$ and $\boldsymbol{b}$ using the dot product operator (·) and the length operator (| |).



14. Briefly explain why || d || == || c || when $d = a \times b$ and || a ||=1.



15. Briefly explain why the measured dynamic range of the same display can differ depending on lighting conditions.

16. A linear function on a plane is $q(x, y) = c_x x + c_y y + c_k$. To interpolate parameters across a triangle we need to find the $c_x, c_y,$ and c_k that define the (unique) linear function that matches the given values $q_{0,} q_{1,} q_2$ at all 3 vertices $(x_{0,} y_0)$, $(x_{1,} y_1)$, $(x_{2,} y_2)$. Derive $c_x, c_y,$ and c_k using the given values. You can use a matrix inversion in the answer.

17. The official projection matrix is given by $\begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

(a) Verify that this matrix scales down all the positions on the far plane.

(b) Explain what is the scale factor, and why is the scale factor designed in such a way.

18. (a) Given two nonparallel, three-dimensional vectors u and v, how can we form an orthogonal coordinate system in which u is one of the basis vectors? Calculate all the basis vectors using the cross product operator (×) and the length operator (| |).

(b) Why is it important that the vectors u,v from (a) be non-parallel?

(c) Given a camera position P, a vector normal to the image plane N, and an up vector Vup, describe how to convert a point W in world coordinates to a point in camera coordinates. Provide your final answer in the form of one (or a product of many) transformation matrix. Hint, the origin in camera coordinates is located at P and the world coordinate axes must be rotated to align with the camera's coordinate axes.