Student #:
Name:

Write down answers in-between questions. Please answer using short sentences.

1. (a) What is the equation of the plane through 3D points $\boldsymbol{a}, \boldsymbol{b}$, and $\boldsymbol{c}$?

(b) What is the normal vector to this plane?

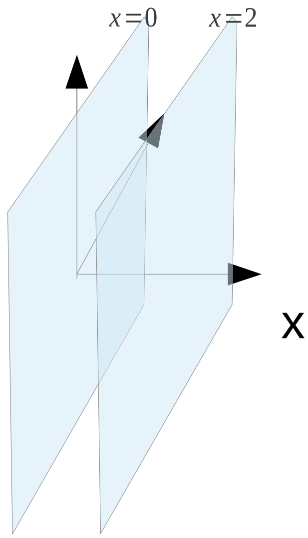
(c) What is the equation of the ray that starts from $\boldsymbol{a}$ toward $\boldsymbol{c}$?

2. Given a rigid transformation matrix R of which the linear part Q is an orthonormal matrix, verify that the inverse of orthormal matrix is as below. (Use the property that the inverse of an orthonormal matrix is its transpose.)

$$R = \begin{bmatrix} Q & \mathbf{u} \\ 0 & 1 \end{bmatrix}, \quad R^{-1} = \begin{bmatrix} Q^T & -Q^T\mathbf{u} \\ 0 & 1 \end{bmatrix}$$

3. What are the ray parameters of the intersection points between ray $\boldsymbol{a}+t\,\boldsymbol{b}$ and a sphere centered at $\boldsymbol{c}$ with radius 1?

4. Calculate the interval of the ray parameters of the intersection points between ray $(3,3,4)+t(-1,-1,-1)$ and a slab defined by two planes shown below.



5. (a) Write down the 4×4 3D matrix to move by (x_m, y_m, z_m).

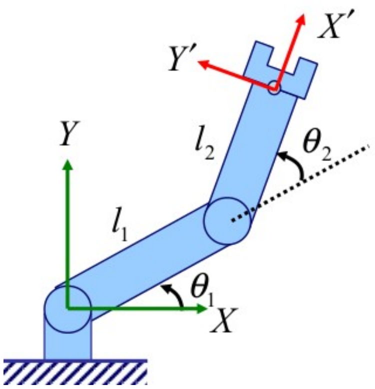
(b) Write down the 4×4 3D matrix to rotate by an angle θ about the z-axis.

(c) Write down the 4×4 rotation matrix M that maps the orthonormal 3D vectors $\boldsymbol{u}=(x_u, y_u, z_u), \boldsymbol{v}=(x_v, y_v, z_v)$, and $\boldsymbol{w}=(x_w, y_w, z_w)$, to orthonormal 3D vectors $\boldsymbol{a}=(x_a, y_a, z_a), \boldsymbol{b}=(x_b, y_b, z_b)$, and $\boldsymbol{c}=(x_c, y_c, z_c)$, so $M\boldsymbol{u}=\boldsymbol{a}, M\boldsymbol{v}=\boldsymbol{b}$, and $M\boldsymbol{w}=\boldsymbol{c}$.

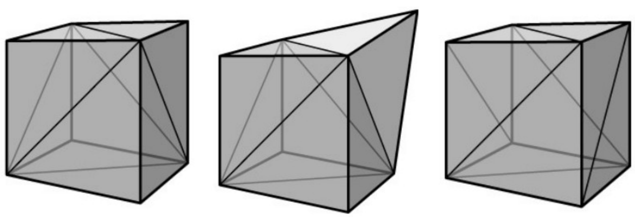
6. Describe in words what this 2D transformation matrix does:

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Write down the transformation matrix T of the tool (at X', Y') in terms of 2D rotation matrix R_θ and 2D translation matrix $T_{(tx,ty)}$. For example, your answer should look something like $T = R_{\theta_3} T_{(0, l_3)}$.

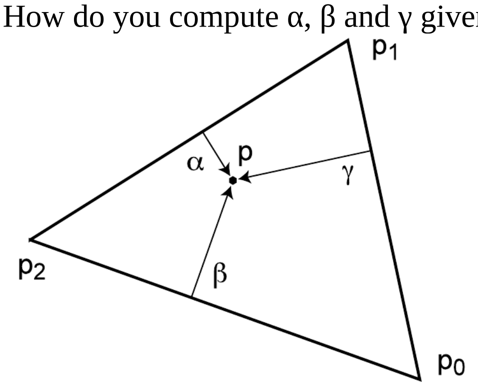


8. Which of these share the same topology? Which share the same geometry?



(a) (b) (c)

9. (a) How do you compute α, β and γ given p1,p2,p0 and p?

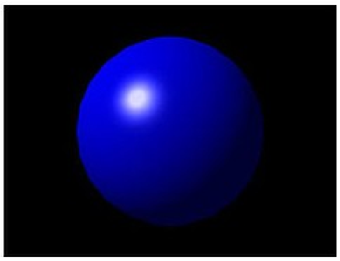
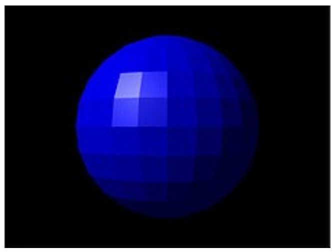
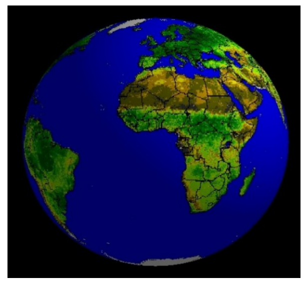


4.

5. $p = \alpha\, p_0 + \beta\, p_1 + \gamma\, p_2$

(b) What are the conditions on α, β and γ for a pixel that is inside the triangle?

10. Look at each of the following images rendered in a pipeline system. For each one, answer the following questions. Describe in words; you don't need to write down any equations. You can assume that the depth test is done automatically after the fragment stage. All three images were generated from the same triangular mesh using the Phong, flat, and gouraud shading techniques, respectively. Only the third image was rendered with a texture map, and the first two images were not. Some attributes you might need include positions, normals, colors, texture coordinates, or scalar values. Write down all the assumptions that you had to make.

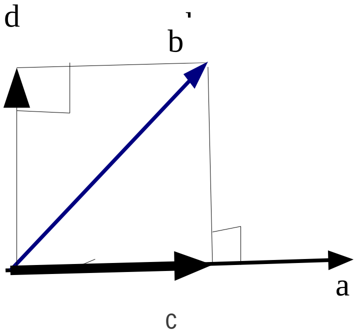
(a) Explain what per-vertex attributes need to be passed from the application to the vertex stage.

(b) Describe the computations that need to be done at the vertex stage.

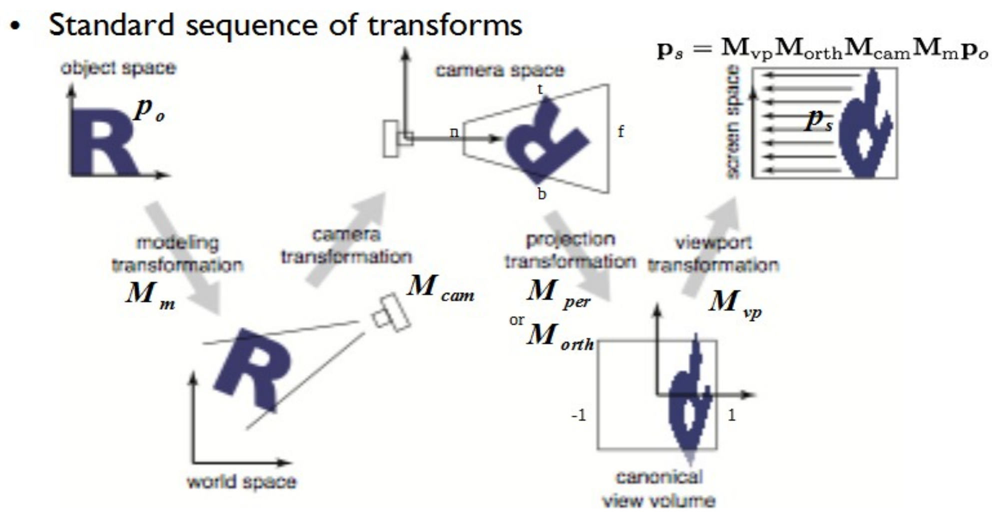
(c) Explain what attributes are interpolated by the rasterizer for the fragment stage.

(d) Describe the computations that need to be done at the fragment stage.

11. Represent vector $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of vector $\boldsymbol{a}$ and $\boldsymbol{b}$ using the dot product operator (·) and the length operator (| |). None of these vectors are unit vectors.



12. Given the matrices $\boldsymbol{M}_{vp}, \boldsymbol{M}_{per}, \boldsymbol{M}_{cam}, \boldsymbol{M}_m$, derive the equation of the ray which corresponds to a screen pixel location (x,y), or in the screen space, $\boldsymbol{p}_s=(x, y, -1, 1)^T$. The ray starts from the near plain toward the far plane. The ray should be defined in the world space. (Assume that $\boldsymbol{M}_{vp}$ preserves the depth z of the canonical view volume).

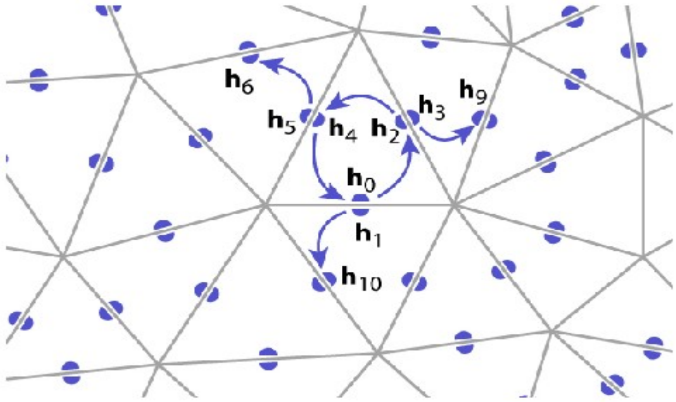


13. Derive the average storage requirement (bytes per vertex) of the indexed triangles representation assuming that a vertex contains a position, a 2D texture coord and a normal (all 4byte float variables) and that the number of triangles is twice the number of vertices on average.

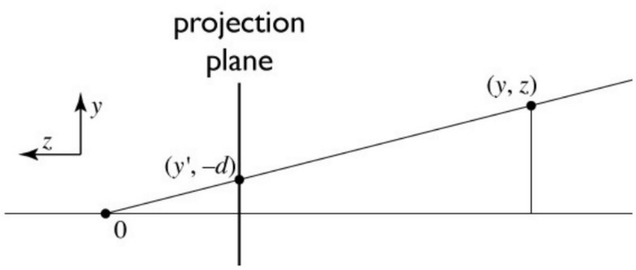
14. Fill in the blanks below of the algorithm that enumerate all edges adjacent to vertex v.

```
EdgesOfVertex(v) {
h = v.h;
do {
        h = ____________;
    } while (____________);
}
```

	pair	next
hedge[0]	1	2
hedge[1]	0	10
hedge[2]	3	4
hedge[3]	2	9
hedge[4]	5	0
hedge[5]	4	6
	⋮	



15. Write down the 3×4 projection matrix that maps a 3d point (x,y,z) to (x',y')?
Hint: similar triangles, homogeneous coordinates



16. Briefly explain why the measured dynamic range of the same display can differ depending on lighting conditions.

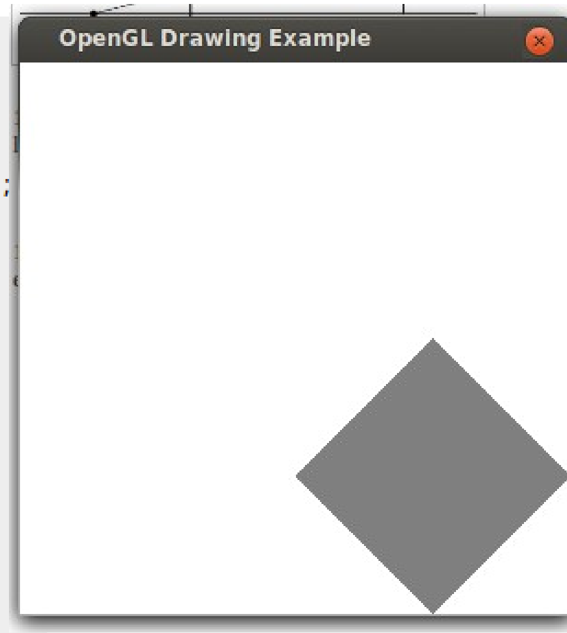
17. [Hidden surface removal] Briefly explain the main downside of the painter's algorithm, and then explain the alternative algorithm that is unanimously used in real-time applications such as games.

18. Fill in the blanks below using glRotatef(angle, axis_x, axis_y, axis_z) and glTranslatef(amount_x, amount_y, amount_z) function.

```
void MyDisplay()
{
    glOrtho(-1.0, 1.0, -1.0, 1.0, -1.0, 1.0);
    glViewport(0, 0, 300, 300);
    glClearColor(1.0, 1.0, 1.0, 1.0);
    glClear(GL_COLOR_BUFFER_BIT|GL_DEPTH_BUFFER_BIT);
    glColor3f(0.5, 0.5, 0.5);
    glLoadIdentity();
    gl________________________
    gl________________________

    double w=sqrt(2*(0.5*0.5))*0.5;
    glBegin(GL_POLYGON);

        glVertex3f(w,w,0);
        glVertex3f(-w,w,0);
        glVertex3f(-w,-w,0);
        glVertex3f(w,-w,0);
    glEnd();
    glFlush();
}
```



19. (a) Given two nonparallel, three-dimensional vectors u and v, how can we form an orthogonal coordinate system in which u is one of the basis vectors? Calculate all the basis vectors using the cross product operator (×) and the length operator (| |).

(b) Why is it important that the vectors u,v from (a) be non-parallel?

(c) Given a camera position P, a vector normal to the image plane N, and an up vector Vup, describe how to convert a point W in world coordinates to a point in camera coordinates. Provide your final answer in the form of one (or a product of many) transformation matrix. Hint, the origin in camera coordinates is located at P and the world coordinate axes must be rotated to align with the camera's coordinate axes.