Student #:
Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

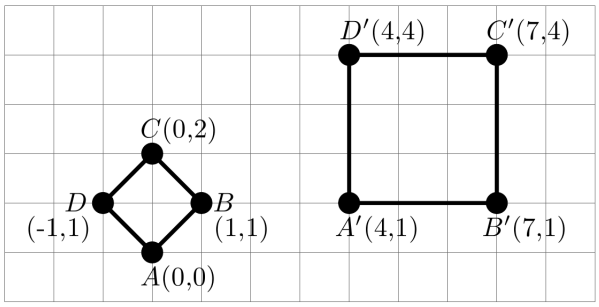
1. (a) Describe an algorithm (other than thresholding) which will convert a grayscale image (8 bits per pixel) to a bi-level black and white image (1 bit per pixel), with the same number of pixels, while retaining as much detail as possible.

(b) Compare and contrast half-toning and error diffusion. Include in your answer an explanation of the situations in which each is superior to the other.

2. (Open GL) We wish to produce two algorithms: one which draws the outline of a circle and one which draws a filled circle.
(a) Describe an algorithm which will draw a one-pixel wide outline of a circle of integer radius, R, centered on the origin.

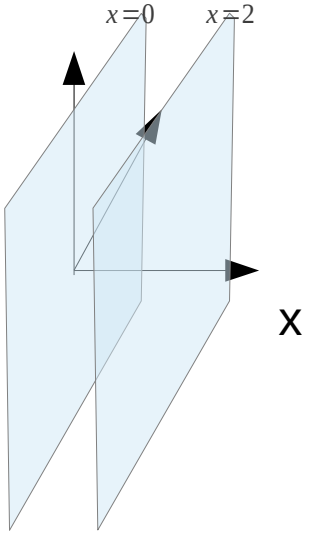
(b) Describe the modifications required to your algorithm to make it draw a filled circle.

3. Give a matrix, or a product of matrices, which will transform the square ABCD to the square A'B'C'D'.

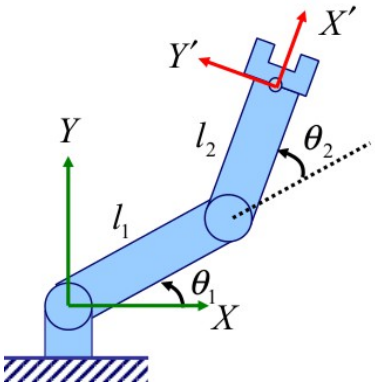


4. What is the equation of the plane through 3D points $\boldsymbol{p_1}, \boldsymbol{p_2}, \boldsymbol{p_3}$? What is the normal vector to this plane?

5. Calculate the interval of the ray parameters of the intersection points between ray $(1,1,1)+ t(-1,-1,-1)$ and a slab defined by two planes shown below.



6. Write down the transformation matrix T of the tool (at X', Y') in terms of 2D rotation matrix R_θ and 2D translation matrix $T_{(tx,ty)}$. For example, your answer should look something like $T = R_{\theta_3} T_{(0,l_3)}$.



7. In ray tracing, once we have determined where a ray strikes an object, the illumination at the intersection point can be calculated using the formula:

$$I = I_a k_a + \sum_i I_i k_d (\boldsymbol{L_i} \cdot \boldsymbol{N}) + I_i k_s (\boldsymbol{R_i} \cdot \boldsymbol{V})^n$$

Explain what real effect each of the three terms is trying to model, how accurately it models the real effect, and explain what each of the following symbols means, within the context of this formula:

I , I_a , i , I_i , k_a , k_d , k_s , L_i , $\boldsymbol{N}$, $\boldsymbol{R_i}$, $\boldsymbol{V}$, $\boldsymbol{n}$

8. (a) Write down the 4×4 3D matrix to move by (x_m, y_m, z_m).

(b) Write down the 4×4 3D matrix to rotate by an angle θ about the y-axis.

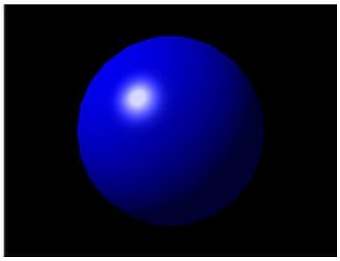
(c) Write down the 4×4 rotation matrix M that maps the orthonormal 3D vectors $\boldsymbol{u}=(x_u, y_u, z_u), \boldsymbol{v}=(x_v, y_v, z_v)$, and $\boldsymbol{w}=(x_w, y_w, z_w)$, to orthonormal 3D vectors $\boldsymbol{a}=(x_a, y_a, z_a), \boldsymbol{b}=(x_b, y_b, z_b)$, and $\boldsymbol{c}=(x_c, y_c, z_c)$, so $M\boldsymbol{u}=\boldsymbol{a}, M\boldsymbol{v}=\boldsymbol{b}$, and $M\boldsymbol{w}=\boldsymbol{c}$.

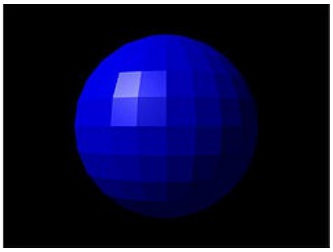
9. (a) Given two nonparallel, three-dimensional vectors u and v, how can we form an orthogonal coordinate system in which u is one of the basis vectors? Calculate all the basis vectors using the cross product operator (×) and the length operator (| |).

(b) Why is it important that the vectors u,v from (a) be non-parallel?

(c) Given a camera position P, a vector normal to the image plane N, and an up vector Vup, describe how to convert a point W in world coordinates to a point in camera coordinates. Provide your final answer in the form of one (or a product of many) transformation matrix. Hint, the origin in camera coordinates is located at P and the world coordinate axes must be rotated to align with the camera's coordinate axes.

10. Look at each of the following images rendered in a pipeline system. For each one, answer the following questions. Describe in words; you don't need to write down any equations. You can assume that the depth test is done automatically after the fragment stage. All three images were generated from the same triangular mesh using the Phong, and flat shading techniques, respectively. Some attributes you might need include positions, normals, colors, texture coordinates, or scalar values. Write down all the assumptions that you had to make.





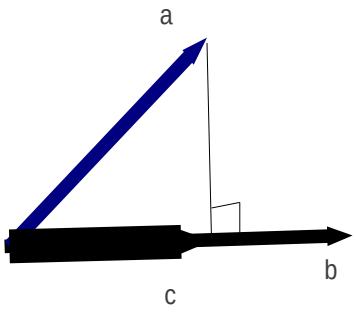
(a) Explain what per-vertex attributes need to be passed from the application to the vertex stage.

(b) Describe the computations that need to be done at the vertex stage.

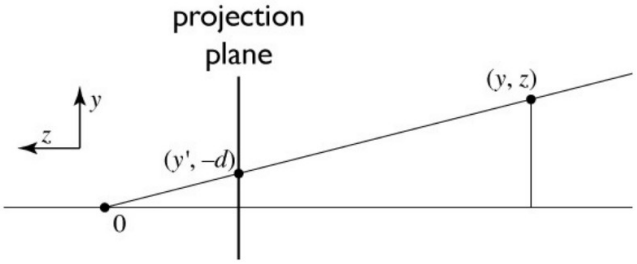
(c) Explain what attributes are interpolated by the rasterizer for the fragment stage.

(d) Describe the computations that need to be done at the fragment stage.

11. Represent vector $\boldsymbol{c}$ in terms of vector $\boldsymbol{a}$ and $\boldsymbol{b}$ using the dot product operator (·) and the length operator (| |).



12. Write down the 3×4 projection matrix that maps a 3d point (x,y,z) to (x',y')?
Hint: similar triangles, homogeneous coordinates



13. Briefly explain why the measured dynamic range of the same display can differ depending on lighting conditions.

14. (Open GL) Create a hierarchical model of the object in figure composed of four cylinders connected at the end. You have available the geometric primitive DrawCylinder(float r, float h), which draws a cylinder of radius r and height h, whose center is the origin and where the height is along the x axis. The animation consists in showing how to move from the position in Figure 1 to the position in Figure 2. Do not use cos or sin functions in the answer.
Hint: you may need some of the following functions.

- glRotate(angle, axisx, axisy, axisz)
- glTranslate(dx, dy, dz)
- glScale(sx, sy, sz)
- glPushMatrix()
- glPopMatrix()

Figure 1.

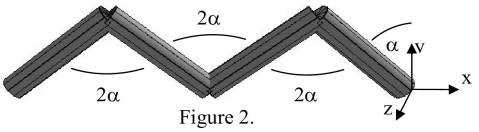
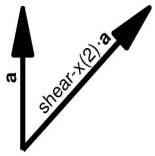


Figure 2.

15. Match each transformation diagram on the right with the corresponding matrix on the left; that is, the matrix should plausibly yield the transform.

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



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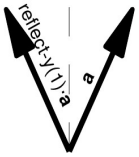
b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



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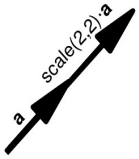
c) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$



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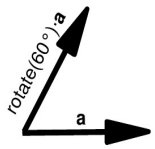
d) $\begin{bmatrix} 0.5 & -0.866 & 0 \\ 0.866 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$



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e) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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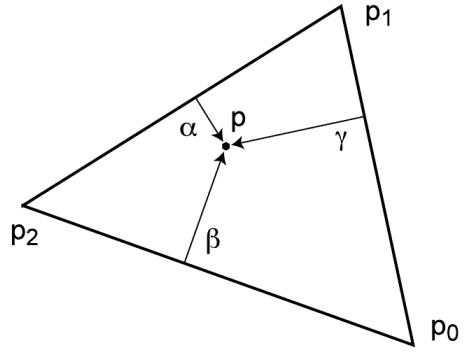
16. (a) What is an orthonormal matrix?

(b) Compute the inverse of the following orthonormal matrix:

$$\begin{bmatrix} 0.71 & 0.0 & -0.71 \\ 0.0 & 1.0 & 0.0 \\ 0.71 & 0.0 & 0.71 \end{bmatrix}$$

17. (a) How do you compute α, β and γ?



4.

5. $p = \alpha\, p_0 + \beta\, p_1 + \gamma\, p_2$

(b) What are the conditions on α, β and γ for a pixel that is inside the triangle?