Student #:
Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

배점 채점기준
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1M : 6점

1. How many bytes are necessary to store a 1024×1024 color image with an alpha channel using 8 bits per channel ?

4 channels ; Red, Green, Blue, Alpha

$1024\times1024\times4\times8$ bits $=1024\times1024\times4$ bytes $=4$MB

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2. What is a parametric form (or explicit equation) for the axis-aligned 2D ellipse of which center is at $\boldsymbol{p}$, width and height are a, b ? (hint: use parameter $t\in[0,2\pi)$. e.g., { f(t) | $t\in[0,2\pi)$ })

$$\left\{ \mathbf{p} + \left(\frac{a}{2}\cos t, \frac{b}{2}\sin t\right) \;\middle|\; t\in[0,2\pi) \right\}$$

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3. What is the implicit equation of the plane through 3D points (1,0,0), (0,1,0), and (0,0,1)? What is the parametric equation? What is the normal vector to this plane?

method 1: $\{(x,y,z) \mid ax+by+cz=1\}$

from 3 constraints
$a\cdot1+b\cdot0+c\cdot0=1$
$a\cdot0+b\cdot1+c\cdot0=1$
$a\cdot0+b\cdot0+c\cdot1=1$
$\rightarrow a=b=c=1$

$\therefore$ implicit equation $=\{(x,y,z) \mid x+y+z=1\}$

normal vector : $(1,1,1)$

method 2: $\{\mathbf{v} \mid (\mathbf{v}-(1,0,0))\cdot \mathbf{n}=0\}$

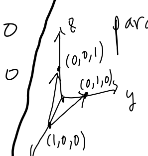
from constraints;
$((0,1,0)-(1,0,0))\cdot\mathbf{n}=0$
$((0,0,1)-(1,0,0))\cdot\mathbf{n}=0$
$|\mathbf{n}|=1$

where $\mathbf{n}=(n_x, n_y, n_z)$

$n_y-n_x=0$
$n_z-n_x=0$
$n_x^2+n_y^2+n_z^2=1$

$\therefore \mathbf{n}=\left(\frac{1}{\sqrt3},\frac{1}{\sqrt3},\frac{1}{\sqrt3}\right)$

$\therefore$ implicit equation $=\left\{\mathbf{v} \mid (\mathbf{v}-(1,0,0))\cdot\left(\frac{1}{\sqrt3},\frac{1}{\sqrt3},\frac{1}{\sqrt3}\right)=0\right\}$



parametric equation:
$\{(1,0,0)+u(-1,1,0)+v(-1,0,1) \mid u,v\in\mathbb{R}\}$

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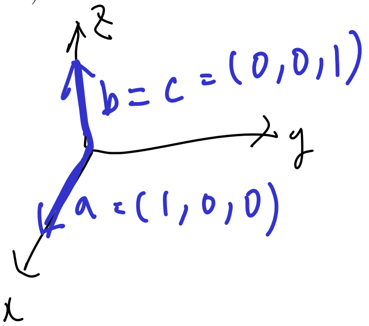
계산 틀리면 8

4. Show by counterexample that is is not always true that for 3D vectors a,b, and c, $\boldsymbol{a}\times(\boldsymbol{b}\times\boldsymbol{c})=(\boldsymbol{a}\times\boldsymbol{b})\times\boldsymbol{c}$.



$b\times c=(0,0,0)$

$\therefore a\times(b\times c)=(0,0,0)$

$a\times b=(0,-1,0)$

$(a\times b)\times c=(-1,0,0)$

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5. What are the ray parameters of the intersection points between ray $(1,1,1)+t(-1,-1,-1)$ and the sphere centered at the origin with radius 1?

t ; $\bar{p}$; $\bar{d}$

$|\bar{p}+t\bar{d}|=1$

$\therefore (\bar{p}+t\bar{d})\cdot(\bar{p}+t\bar{d})=1$

$\underbrace{\bar{p}\cdot\bar{p}}_{=3} + 2\underbrace{\bar{d}\cdot\bar{p}}_{=-3}\,t + \underbrace{\bar{d}\cdot\bar{d}}_{=3}\,t^2 = 1$

$3t^2-6t+2=0$

$\therefore t=\dfrac{6\pm\sqrt{6^2-4\cdot3\cdot2}}{6}$; $6^2-4\cdot3\cdot2 = 12 = 2^2\cdot3$

$=\dfrac{3\pm\sqrt3}{3}$

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6. (a) Write down the 4×4 3D matrix to move by (x_m, y_m, z_m).

$$T\begin{pmatrix}x\\y\\z\\1\end{pmatrix}=\begin{pmatrix}x+x_m\\y+y_m\\z+z_m\\1\end{pmatrix}\quad\therefore T=\begin{pmatrix}1&0&0&x_m\\0&1&0&y_m\\0&0&1&z_m\\0&0&0&1\end{pmatrix}$$

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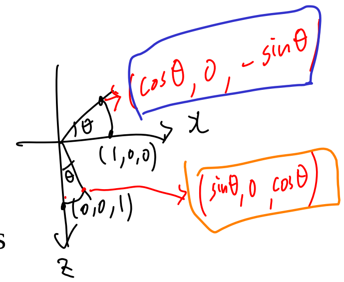
(b) Write down the 4×4 3D matrix to rotate by an angle θ about the y-axis.

$$R\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}=\begin{pmatrix}\boxed{\begin{matrix}\cos\theta\\0\\-\sin\theta\\0\end{matrix}}&\begin{matrix}0\\1\\0\\0\end{matrix}&\boxed{\begin{matrix}\sin\theta\\0\\\cos\theta\\0\end{matrix}}&\begin{matrix}0\\0\\0\\1\end{matrix}\end{pmatrix}\quad\therefore R=\begin{pmatrix}\cos\theta&0&\sin\theta&0\\0&1&0&0\\-\sin\theta&0&\cos\theta&0\\0&0&0&1\end{pmatrix}$$



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(c) Write down the 4×4 rotation matrix M that maps the orthonormal 3D vectors $\boldsymbol{u}=(x_u, y_u, z_u), \boldsymbol{v}=(x_v, y_v, z_v)$, and $\boldsymbol{w}=(x_w, y_w, z_w)$, to orthonormal 3D vectors $\boldsymbol{a}=(x_a, y_a, z_a), \boldsymbol{b}=(x_b, y_b, z_b)$, and $\boldsymbol{c}=(x_c, y_c, z_c)$, so $M\boldsymbol{u}=\boldsymbol{a}, M\boldsymbol{v}=\boldsymbol{b}$, and $M\boldsymbol{w}=\boldsymbol{c}$.

$$M\begin{pmatrix}u^T&v^T&w^T&\mathbf{0}\\0&0&0&1\end{pmatrix}=\begin{pmatrix}a^T&b^T&c^T&\mathbf{0}\\0&0&0&1\end{pmatrix}\quad\therefore M=\begin{pmatrix}x_a&x_b&x_c&0\\y_a&y_b&y_c&0\\z_a&z_b&z_c&0\\0&0&0&1\end{pmatrix}\cdot\begin{pmatrix}x_u&x_v&x_w&0\\y_u&y_v&y_w&0\\z_u&z_v&z_w&0\\0&0&0&1\end{pmatrix}^{-1}$$

7. Describe in words what this 2D transformation matrix does:

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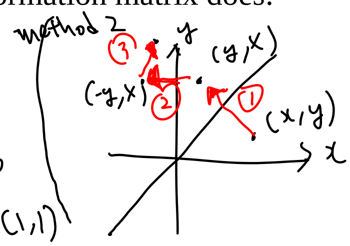
$$\begin{bmatrix}0&-1&1\\1&0&1\\0&0&1\end{bmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix}=\begin{pmatrix}-y+1\\x+1\\1\end{pmatrix}$$

method 1: $\begin{pmatrix}\cos90^\circ&-\sin90^\circ&1\\\sin90^\circ&\cos90^\circ&1\\0&0&1\end{pmatrix}$ ∴ ① rotate by 90° ② Translate by (1,1)

method 2



① mirror about y=x
② mirror about x=0
③ translate by (1,1)

8. Derive the incremental form of the midpoint line-drawing algorithm for $0<m\leq1$.
(hint: modify the non-incremental form shown below.)

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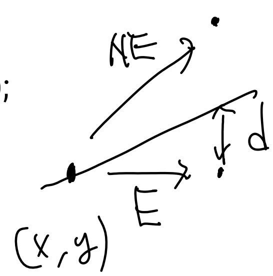
```
double x=ceil(x0);
while (x<= floor(x1))
{
   double y=b+m*x;
   output(x, round(y));
   x=x+1.0;
}
```




```
int x = ceil(x0);
int y = round(b+m*x);
double d = b+m(x+1)
while (x <= floor(x1))
{
    if ( d > 0.5)
    {
        y += 1;
        d -= 1;
    }
    x += 1;
    d += m;
    output(x, y);
}
```

9. Which of these share the same topology? Which share the same geometry?

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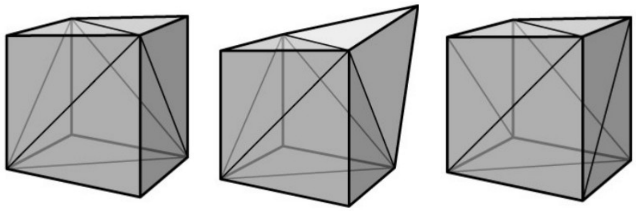
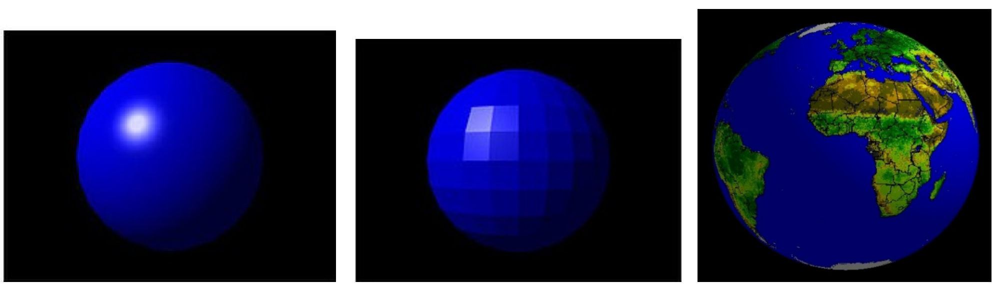


(a) (b) (c)

(a), (b) ; Same topology

(a), (c) ; Same geometry

10. Look at each of the following images rendered in a pipeline system. For each one, answer the following questions. Describe in words; you don't need to write down any equations. You can assume that the depth test is done automatically after the fragment stage. All three images were generated from the same triangular mesh using the Phong, flat, and gouraud shading techniques, respectively. Some attributes you might need include positions, normals, colors, texture coordinates, or scalar values. Write down all the assumptions that you had to make.



① ② ③

(a) Explain what per-vertex attributes need to be passed from the application to the vertex stage.

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① : position, normal, color

② : 〃

③ : 〃 , texture coordinates

(b) Describe the computations that need to be done at the vertex stage.

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① transform position, normal,

② 〃 , shading

③ 〃 , 〃

(c) Explain what attributes are interpolated by the rasterizer for the fragment stage.

① normal, color

② shaded color

③ 〃 , texture coordinates

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(d) Describe the computations that need to be done at the fragment stage.

① interpolate color, normal, calculate shading,

② interpolate shaded color

③ 〃 , texture coordinates,

sample textures, combine colors

$|a| \cdot \cos\theta =$

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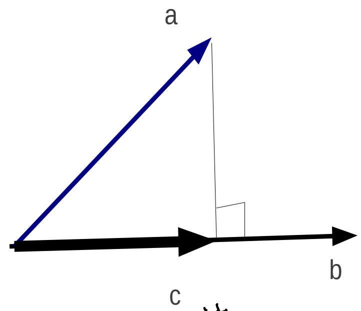
11. Represent vector $\boldsymbol{c}$ in terms of vector $\boldsymbol{a}$ and $\boldsymbol{b}$ using the dot product operator (·) and the length operator (| |).



$$|c| = a \cdot \frac{b}{|b|}$$

$$c = |c| \cdot \frac{b}{|b|} = \left(a \cdot \frac{b}{|b|}\right) \cdot \frac{b}{|b|}$$

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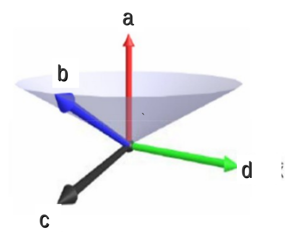
12. Represent unit vector $\boldsymbol{c}$ and $\boldsymbol{d}$ in terms of vector $\boldsymbol{a}$ and $\boldsymbol{b}$ using the cross product operator (×) and the length operator (| |). Vector $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ are in the same plane, and $\boldsymbol{d}$ is orthogonal to the other vectors.



$$\bar{d} = \frac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}$$

$$\bar{c} = \bar{d} \times \frac{\bar{a}}{|\bar{a}|} = \left(\frac{\bar{a} \times \bar{b}}{|\bar{a} \times \bar{b}|}\right) \times \frac{\bar{a}}{|\bar{a}|}$$

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13. Derive the average storage requirement (bytes per vertex) of the indexed triangle set representation assuming that a vertex contains a position and a normal (4byte float variables) and that the number of triangles is twice the number of vertices on average.

36 bytes

vertex buffer: $(3+3) \cdot 4$ bytes per vertex + = 48 bytes/vertex

position normal x,y,z

522 index buffer: 12 bytes per triangle ≒ 24 bytes per vertex

12 3·4

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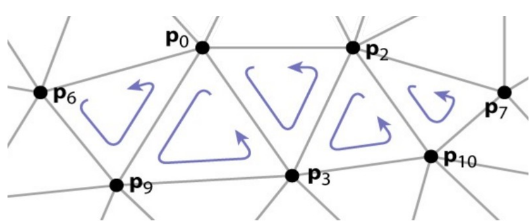
14. Write down the array of vertex indices that represents the following triangle strip consisting of 5 triangles. What is the advantage of the triangle strip representation?



(6, 9, 0, 3, 2, 10, 7)

efficient memory usage, faster rendering

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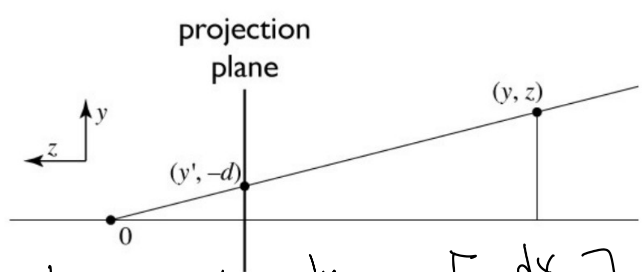
15. Write down the 3×4 projection matrix that maps a 3d point (x,y,z) to (x',y')?
Hint: similar triangles, homogeneous coordinates



$$\frac{y'}{y} = \frac{-d}{z} \quad \therefore \quad y' = -\frac{dy}{z}, \quad x' = -\frac{dx}{z}$$

$$\begin{bmatrix} -\frac{dx}{z} \\ -\frac{dy}{z} \\ 1 \end{bmatrix} \sim \begin{bmatrix} dx \\ dy \\ -z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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16. Briefly explain why the measured dynamic range of the same display can differ depending on lighting conditions.

$R_d = \dfrac{I_{max} + k}{I_{min} + k}$

viewing flare k: light reflected by the display depends on lighting conditions.
e.g. k=0 in a dark room

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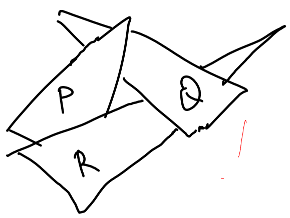
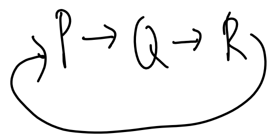
17. [Hidden surface removal] Briefly explain the main downside of the painter's algorithm, and then explain the alternative algorithm that is unanimously used in real-time applications such as games.

if there are cycles, there is no sort of the graph of occlusions.





Z-buffer keeps track of closest depth so far