

Student #:

Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

1. How many bytes are necessary to store a 1024×1024 color image with an alpha channel using 8 bits per channel ?
2. What is a parametric form (or explicit equation) for the axis-aligned 2D ellipse of which center is at p , width and height are a, b ? (hint: use parameter $t \in [0, 2\pi)$. e.g., $\{ f(t) \mid t \in [0, 2\pi) \}$)
3. What is the implicit equation of the plane through 3D points $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$? What is the parametric equation? What is the normal vector to this plane?
4. Show by counterexample that it is not always true that for 3D vectors a, b , and c ,
$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \times \mathbf{c}.$$
5. What are the ray parameters of the intersection points between ray $(1,1,1) + t(-1,-1,-1)$ and the sphere centered at the origin with radius 1?

6. (a) Write down the 4×4 3D matrix to move by (x_m, y_m, z_m) .

(b) Write down the 4×4 3D matrix to rotate by an angle θ about the y-axis.

(c) Write down the 4×4 rotation matrix M that maps the orthonormal 3D vectors $\mathbf{u} = (x_u, y_u, z_u)$, $\mathbf{v} = (x_v, y_v, z_v)$, and $\mathbf{w} = (x_w, y_w, z_w)$, to orthonormal 3D vectors $\mathbf{a} = (x_a, y_a, z_a)$, $\mathbf{b} = (x_b, y_b, z_b)$, and $\mathbf{c} = (x_c, y_c, z_c)$, so $M\mathbf{u} = \mathbf{a}$, $M\mathbf{v} = \mathbf{b}$, and $M\mathbf{w} = \mathbf{c}$.

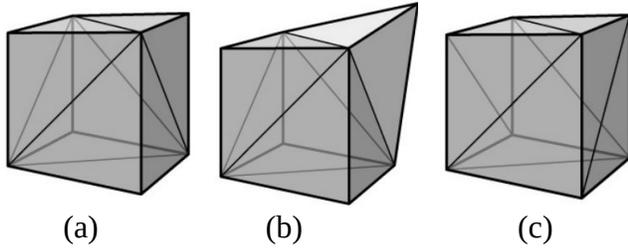
7. Describe in words what this 2D transformation matrix does:

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

8. Derive the incremental form of the midpoint line-drawing algorithm for $0 < m \leq 1$.
(hint: modify the non-incremental form shown below.)

```
double x=ceil(x0);
while (x<= floor(x1))
{
    double y=b+m*x;
    output(x, round(y));
    x=x+1.0;
}
```

9. Which of these share the same topology? Which share the same geometry?



10. Look at each of the following images rendered in a pipeline system. For each one, answer the following questions. Describe in words; you don't need to write down any equations. You can assume that the depth test is done automatically after the fragment stage. All three images were generated from the same triangular mesh using the Phong, flat, and gouraud shading techniques, respectively. Some attributes you might need include positions, normals, colors, texture coordinates, or scalar values. Write down all the assumptions that you had to make.



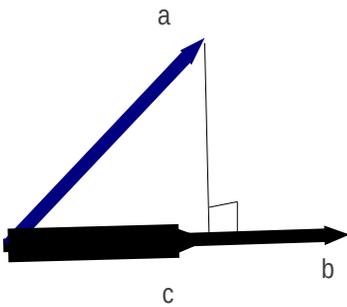
(a) Explain what per-vertex attributes need to be passed from the application to the vertex stage.

(b) Describe the computations that need to be done at the vertex stage.

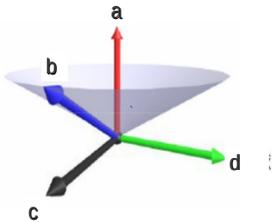
(c) Explain what attributes are interpolated by the rasterizer for the fragment stage.

(d) Describe the computations that need to be done at the fragment stage.

11. Represent vector c in terms of vector a and b using the dot product operator (\cdot) and the length operator ($|\cdot|$).

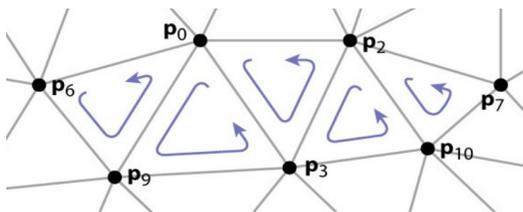


12. Represent vector c and d in terms of vector a and b using the cross product operator (\times) and the length operator ($|\cdot|$). Vector a, b, c are in the same plane, and d is orthogonal to the other vectors. All vectors have unit length.

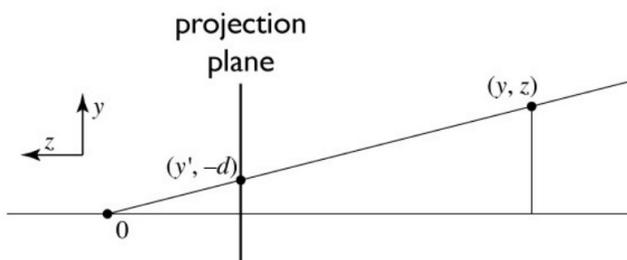


13. Derive the average storage requirement (bytes per vertex) of the indexed triangle set representation assuming that a vertex contains a position and a normal (4byte float variables) and that the number of triangles is twice the number of vertices on average.

14. Write down the array of vertex indices that represents the following triangle strip consisting of 5 triangles. What is the advantage of the triangle strip representation?



15. Write down the 3×4 projection matrix that maps a 3d point (x, y, z) to (x', y') ?
Hint: similar triangles, homogeneous coordinates



16. Briefly explain why the measured dynamic range of the same display can differ depending on lighting conditions.

17. [Hidden surface removal] Briefly explain the main downside of the painter's algorithm, and then explain the alternative algorithm that is unanimously used in real-time applications such as games.