Student \#:
Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

1. (a) If the following 1 D discrete filters are used to define 2 D filters and applied to images, which filter goes with which operation? hints: assuming $a[i, j]=a_{1}[i] a_{1}[j]$, a filtering operation can be defined using the convolution operator * as follows:
$(a * b)[i, j]=\sum_{i^{\prime}, j^{\prime}} a\left[i^{\prime}, j^{\prime}\right] b\left[i-i^{\prime}, j-j^{\prime}\right]=\sum_{i^{\prime}} a_{1}\left[i^{\prime}\right]\left(\sum_{j^{\prime}} a_{1}\left[i^{\prime}\right] b\left[i-i^{\prime}, j-j^{\prime}\right]\right)$, where 1D
filter $a_{1}$ is one of the followings.
2. 


(a) sharpen
(b) blur

(c) differentiate (in some way)
(d) shift right and up
3.


0

(b) Calculate the convolution result of two 1-D signals: $[001110110]$ and $\left[\begin{array}{lll}-1 & 0 & 1\end{array}\right]$.
2. (a) Briefly dscribe the problems of magnification and minification in texture-mapping.
(b) Briefly describe algorithms to reduce the artifacts caused by the minification (two algorithms) and magnification (one algorithm) of textures.
(c) Briefly describe the tri-linear interpolation algorithm.
3. (a) If we use the following 1D reconstruction filter $a(x)$ to reconstruct a continuous function $g(x)$ from a sequence of samples $f[i]$ using continuous-discontinuous convolution, how will the resulting function look like? (Draw $g(x)$ )
hints: the reconstructed function $g(x)$ is defined as $g(x)=\sum_{i} f[i] a(x-i)$ for a arbitrary sequence of samples $f[i]$.
$a(x)$ :


$$
g(x) \text { ? }
$$

f: [ 1,3,1,2,3 ]
(b) Draw a filter that will generate a $C^{1}$ continuous function interpolating $f[i]$.
4. Create a transformation matrix of the iguana in the following figure to locate it into the desired configuration. You have the available set of matrices you can use in the answer.


a. Iguana is facing slightly upward.
$\qquad$ ?

b. Iguana's forward direction is parallel to the ground plane.
$\qquad$

Hint: the last three coordinate frames share the same (local) origin.
5. Consider the second degree polynomial $p(u)=c_{0}+c_{1} u+c_{2} u^{2}=\boldsymbol{u}^{T} \boldsymbol{c}=\left[1 u u^{2}\right]\left[c_{0} c_{1} c_{2}\right]^{T}$ and the control point $\quad \boldsymbol{p}=\left[p_{0} p_{1} p_{2}\right]^{T}$. Given the following set of constraints, describe how to calculate the unknown coefficients $c_{0, c_{1}, c_{2}}$ in terms of a known set of values $a, b, c$.

Constraints:

$$
\begin{aligned}
& p(0)=a \\
& p(1)=b \\
& p^{\prime}(0)=c
\end{aligned}
$$

6. Bezier polynomials can be rendered efficiently with recursive subdivision. It is common to convert a non-Bezier polynomial to an equivalent Bezier polynomial in order to use these rendering techniques. Describe how to do this mathmatically. (Assume that the basis matrices $M_{\text {bezier }}$, and $M_{\text {non-bezier }}$ is known.)
(b) conversion to Beziers
(a) recursive subdivision.
7. Below are four curves and their "control points/polygon." Two of the control polygons are the Bezier control polygon for the curve drawn with it; the other four are not. Indicate which of the control polygons are Bezier control polygons for the corresponding curve and which are not. Justify your answer for the control polygons that are not Bezier control polygons. You may assume that none of the control points overlap or are repeated.
a.

b.

C.

d.

8. Given two matrices cow2wld and wld2cam, a cow can be transformed to the camera space using wld2cam * cow2wld.
Here, cow2wld which transfrom a cow from the object coordinates to the world coordinates, wld2cam which transform a cow from the world coordinates to the camera coordinatesr. You can use a.R, a.T to denote the rotation and translation part of the matrix, that is, $\mathrm{a}=\mathrm{a} . \mathrm{T} * \mathrm{a} . \mathrm{R} . \mathrm{Rx}$ is the rotation matrix along X -axis.
a) describe how to rotate a cow along X-axis in the object space
b) describe how to rotate the cow along the X -axis of the world space.
c) describe how to rotate the cow along the X -axis of the camera space.
9. Suppose we have an image of a gray elephant, with an alpha matte to delineate foreground from background. The image is stored with non-premultiplied alpha (unusually).

a. If we accidentally use the image in a program that expects premultiplied alpha but without premultiplying the alpha values, will the partially transparent edges come out too dark, about right, or too light if the background is:
(a) solid black
(b) solid white
(c) about the same color as the elephant

Explain why by writing down the equations for the correct and incorrect results.
Hints: when using premultiplied alpha $\begin{gathered}\mathrm{C}=\mathrm{A} \text { over } \mathrm{B} \text { becomes } \\ c_{C}^{\prime}=c_{A}^{\prime}+\left(1-\alpha_{A}\right) c_{B}^{\prime}\end{gathered}$
10. The tree below represents a quadtree subdivision of a square. The left most branch is the upper left quarter, the next branch is the upper right, the third branch is the lower left, and the right most branch is the bottom left of a region. The leaf nodes are labeled A-J.
Subdivide the square below to indicate the subdivision of the quad tree. Label each portion of the square with the appropriate label from the tree.

11. Shadow maps are one method of generated shadows using OpenGL. Describe how to implement shadow maps
12. EdgesOfVertex is a function that iterates through the list of half edges adjacent to vertex v . Fill in the blanks (???).

```
EdgesOfVertex(v) \{
```

$\mathrm{h}=\mathrm{v} . \mathrm{h}$;
do \{
$\mathrm{h}=$
???
\} while ( n ! $=$ v.n);
\}

|  | pair | next |
| :--- | :--- | :--- |
| hedge[0] | 1 | 2 |
| hedge[1] | 0 | 10 |
| hedge[2] | 3 | 4 |
| hedge[3] | 2 | 9 |
| hedge[4] | 5 | 0 |
| hedge[5] | 4 | 6 |
|  | $\vdots$ |  |
|  |  |  |



HEdge \{
HEdge pair, next;
Vertex v;
Face f;
\}
13. Derive the average storage requirement (bytes per vertex) of the indexed triangles representation assuming that a vertex contains a position, a 2D texture coord and a normal (all 4byte float variables) and that the number of triangles is twice the number of vertices on average.

