Student \#:
Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

1. (a) If the following 1 D discrete filters are used to define 2 D filters and applied to images, which filter goes with which operation? hints: assuming $a[i, j]=a_{1}[i] a_{1}[j]$, a filtering operation can be defined using the convolution operator * as follows:
$(a * b)[i, j]=\sum_{i^{\prime}, j^{\prime}} a\left[i^{\prime}, j^{\prime}\right] b\left[i-i^{\prime}, j-j^{\prime}\right]=\sum_{i^{\prime}} a_{1}\left[i^{\prime}\right]\left(\sum_{j^{\prime}} a_{1}\left[i^{\prime}\right] b\left[i-i^{\prime}, j-j^{\prime}\right]\right)$, where 1D
filter $\quad a_{1}$ is one of the followings.
2. 


(a) sharpen
(b) blur

(c) differentiate (in some way)
(d) shift right and up
3.


0

(b) Calculate the convolution result of two 1-D signals: $\left[\begin{array}{llllll}0 & 1 & 1 & 0 & 1 & 1\end{array} 0\right.$ ] and $[-101]$.
2. Briefly explain what is moire' pattern, what causes it, and how to prevent it.
3. (a) Briefly dscribe the problems of magnification and minification in texture-mapping.
(b) Briefly describe algorithms to reduce the artifacts caused by the minification (two algorithms) and magnification (one algorithm) of textures.
(c) Briefly describe the tri-linear interpolation algorithm.

4 (a) If we use the following 1D reconstruction filter $a(x)$ to reconstruct a continuous function $g(x)$ from a sequence of samples $f[i]$ using continuous-discontinuous convolution, how will the resulting function look like? (Draw $g(x)$ )
hints: the reconstructed function $g(x)$ is defined as $g(x)=\sum_{i} f[i] a(x-i)$ for a arbitrary sequence of samples $f[i]$.

$$
\begin{array}{llllll}
a(x): \\
& \\
-2 & -1 & 0 & 1 & 2
\end{array}
$$

$\mathrm{f}:$ [ 2,3,1,2,3 ]
(b) Draw a filter that will generate a $C^{1}$ continuous function interpolating $f[i]$.

5 Create a hierarchical model of the object in figure composed of four cylinders connected at the end. You have available the geometric primitive DrawCylinder(float $r$, float $h$ ), which draws a cylinder of radius $r$ and height $h$, whose center is the origin and where the height is defined along the -x axis. (For example, DrawCylinder(r,h) draws the right-most segment of the pipe in Figure 1.)
Write a pseudo code that draws Figure 2 by moving from the position in Figure 1 to the position in Figure 2. (Hint: use a chain of matrix multiplications)


Figure 1.


The other functions you may need are:

```
glRotate(angle, axisx, axisy, axisz)
glTranslate(dx,dy,dz)
glPushMatrix()
glPopMatrix()
```

6. Consider the second degree polynomial $p(u)=c_{0}+c_{1} u+c_{2} u^{2}=\boldsymbol{u}^{T} \boldsymbol{c}=\left[1 u u^{2}\right]\left[c_{0} c_{1} c_{2}\right]^{T}$ and the control point $\quad \boldsymbol{p}=\left[\begin{array}{lll}p_{0} & p_{1} & p_{2}\end{array}\right]^{T}$. Given the following set of constraints, describe how to calculate the unknown coefficients $c_{0,} c_{1}, c_{2}$ in terms of a known set of values $a, b, c$.

Constraints:

$$
\begin{aligned}
& p(0)=a \\
& p(1)=b \\
& p^{\prime}(0)=c
\end{aligned}
$$

7. Bezier polynomials can be rendered efficiently with recursive subdivision. It is common to convert a non-Bezier polynomial to an equivalent Bezier polynomial in order to use these rendering techniques. Describe how to do this mathmatically. (Assume that the basis matrices $M_{\text {bezier }}$, and $M_{\text {non-bezier }}$ is known.)
(b) conversion to Beziers
(a) recursive subdivision.
8. Briefly describe the Phong shading method discussing how it integrates in the modern graphics pipeline, its advantages and disadvantages.
9. Write down a sequence of numbers in the order that the k-d tree is traversed when checking intersections between a ray and the scene. Every non-leaf node can be thought of as implicitly generating a splitting hyperplane that divides the space into two bounding boxes. The numbers at the leaf nodes represents the intersection test between the ray and the primitives. The number at the arrow represents the intersection test between the ray and the bounding box. Omit numbers that correspond to pruned operations (that is not executed). Do not exclude intersection tests that fails; for example, the correct answer is " 1 " for a ray that doesn't collide with the outer-most bounding box.


Ray 1 :

Ray 2
10. Below are four curves and their "control points/polygon." Two of the control polygons are the Bezier control polygon for the curve drawn with it; the other four are not. Indicate which of the control polygons are Bezier control polygons for the corresponding curve and which are not. Justify your answer for the control polygons that are not Bezier control polygons. You may assume that none of the control points overlap or are repeated.
a.

b.

C.

d.

11. Describe the algorithm used to determine whether a ray intersects a polygon.
12. Given two matrices cow2wld and wld2cam, a cow can be transformed to the camera space using wld2cam * cow2wld.
Here, cow2wld which transfrom a cow from the object coordinates to the world coordinates, wld2cam which transform a cow from the world coordinates to the camera coordinatesr. You can use a.R, a.T to denote the rotation and translation part of the matrix, that is, $a=a . T$ * a.R. Rx is the rotation matrix along X -axis.
a) describe how to rotate a cow along X -axis in the object space
b) describe how to rotate the cow along the X-axis of the world space.
c) describe how to rotate the cow along the X -axis of the camera space.
13. Suppose we have an image of a gray elephant, with an alpha matte to delineate foreground from background. The image is stored with non-premultiplied alpha (unusually).

a. If we accidentally use the image in a program that expects premultiplied alpha but without premultiplying the alpha values, will the partially transparent edges come out too dark, about right, or too light if the background is:
(a) solid black
(b) solid white
(c) about the same color as the elephant

Explain why by writing down the equations for the correct and incorrect results.
Hints: when using premultiplied alpha $\begin{gathered}C=A \text { over } B \text { becomes } \\ c_{C}^{\prime}=c_{A}^{\prime}+\left(1-\alpha_{A}\right) c_{B}^{\prime}\end{gathered}$
14. (a) Construct a summed area table from the texture below, and explain how you can calculate the average value of the middle 4 pixels of the texture using the summed area table.

(b) How is this related to texture filtering?
15. (a) Suppose that we have two coordinate systems: one is the reference coordinate system with its origin at $\quad P_{0}$ and three standard bases $u, v, w$, The other coordinate system has its origin at $P_{1}=\left(P_{x}, P_{y}, P_{z}\right)$ and three orthonormal vectors $U, V, W$. Find a $4 x 4$ homogeneous matrix that transforms the coordinate of a point in the other system into the coordinate with respect to the reference coordinate system.

(b) Find a $4 \times 4$ homogeneous matrix that transforms the coordinate of a point in the reference system into the coordinate with respect to the other coordinate system without using the matrix inversion operation. (Hint: the inverse of an orthonormal matrix is its transpose.)

