Student #:
Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

1. The image below is an image of a 3 pixel thick vertical line.
(a) Show the resulting image obtained after convolution of the original with the following approximation of the derivative filter [−1, 0, 1] in the horizontal direction.
Hint: the origin of a signal is the center element.

1-D convolution: $(f * g)[n] \stackrel{\text{def}}{=} \sum_{m=-\infty}^{\infty} f[m]\, g[n-m]$

0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0
0 0 0 1 1 1 0 0 0

(b) Suggest a filter which when convolved with the same image would yield a single maximum in the middle of the line. Demonstrate the result of the convolution on the original image.

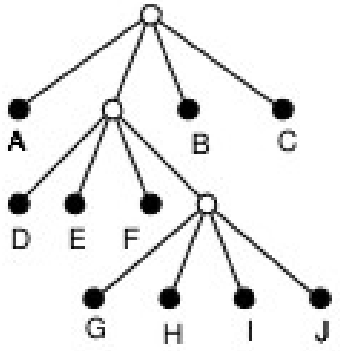
2. Biefly explain what is moire' pattern, what causes it, and how to prevent it.

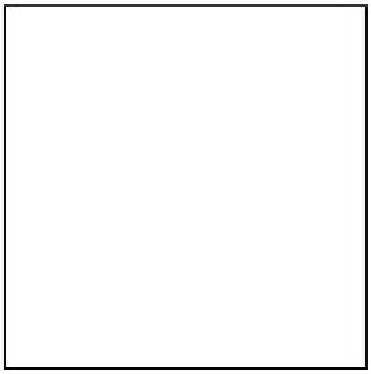
3. (a) Briefly dscribe the problems of magnification and minification in texture-mapping.

(b) Briefly describe algorithms to reduce the artifacts caused by the minification (two algorithms) and magnification (one algorithm) of textures.

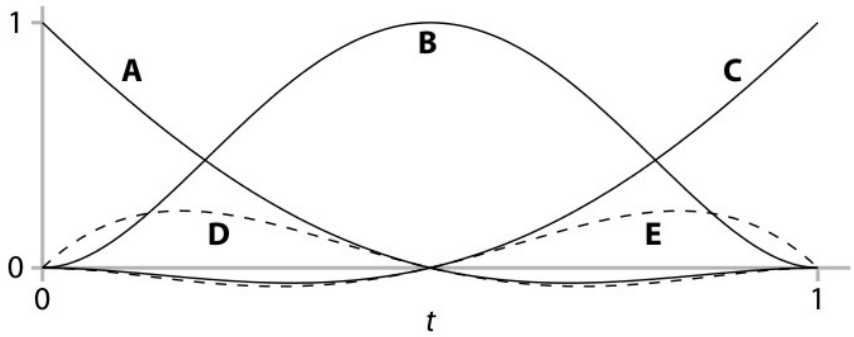
4. Consider the second degree polynomial $p(u)=c_0+c_1u+c_2u^2=\boldsymbol{u}^T\boldsymbol{c}=[1\,u\,u^2][c_0\,c_1\,c_2]^T$ and the control point $\boldsymbol{p}=[\,p0\;p1\;p2]^T$. Derive the matrix A so that the coefficients $\boldsymbol{c}=A^{-1}\boldsymbol{p}$ make p(u) interpolate all control points (for $u\in[0,1], u_0=0, u_1=0.5, u_2=1$).

5. The tree below represents a quadtree subdivision of a square. The left most branch is the upper left quarter, the next branch is the upper right, the third branch is the lower left, and the right most branch is the bottom left of a region. The leaf nodes are labeled A-J.
Subdivide the square below to indicate the subdivision of the quad tree. Label each portion of the square with the appropriate label from the tree.





6. When you look at the existing spline code, you see that it is using a sequence of segments, each defined by a quartic (fourth degree) spline with an unfamiliar spline matrix. You plot the basis functions and you see the following curves:



(a) Which of the five control points (arbitrarily labeled A through E in the plot) will the curve pass through, and for what values of t?

(b) Which control points affect the tangent to the curve at t = 0? At t = 0.5? At t = 1?

(c) Does this spline have the convex hull property? How did you tell?

7. Describe the algorithm used to determine whether a ray intersects a polygon.

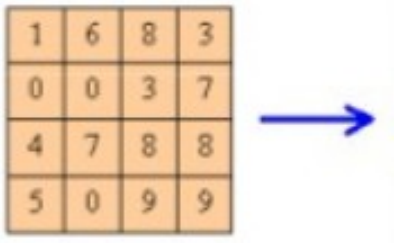
8. The standard process for dealing with translucent objects involves disabling writes to the depth buffer and sorting transparent objects and/or polygons based on distance to the camera.
a) Explain why writes to the Z-buffer should be disabled.

b) Explain why sorting is necessary using the following equation.

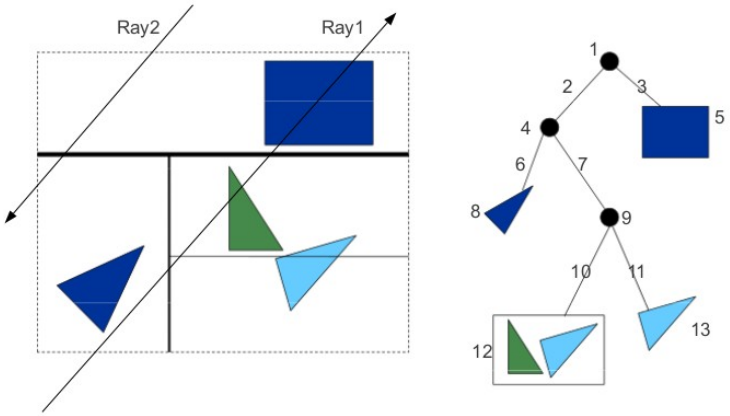
C = A **over** B becomes

$$c'_C = c'_A + (1 - \alpha_A)c'_B$$

9. Construct a summed area table from the texture below, and explain how you can calculate the average value of the bottom half of the texture using the summed area table.



10. Write down a sequence of numbers in the order that the k-d tree is traversed when checking intersections between a ray and the scene. Every non-leaf node can be thought of as implicitly generating a splitting hyperplane that divides the space into two parts. The numbers at the leaf nodes represents the intersection test between the ray and the primitives. The number at the arrow represents the tree traversal. Omit numbers that correspond to pruned operations (that is not executed). Do not exclude intersection tests that fails; for example, the correct answer is “1” for a ray that doesn't collide with the hyperplane 1.



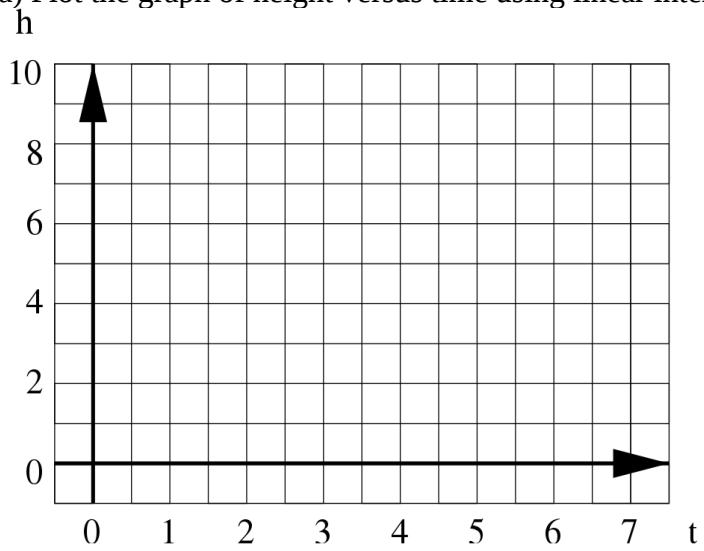
Ray 1:

Ray 2

11. What is the difference between Bump Mapping and Displacement Mapping? Describe a situation where you can see the difference.

12. Suppose we wish to animate a bouncing ball. We are given keyframes for the ball's motion. The key frames are at times $T_f = \{0,1,2,3,4,5,6\}$, with associated height values of $H_f = \{0,5,8,9,8,5,0\}$.

a) Plot the graph of height versus time using linear interpolation between key frames



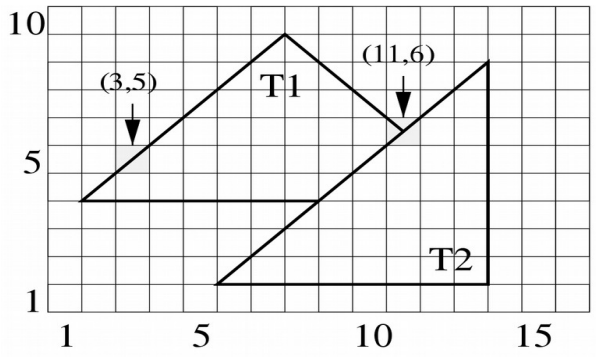
b) Give an equation for the height h(t) over the interval t=4...5 using linear interpolation.

c) In general, suppose we are given times $T_f = \{t_0, t_1\}$, and associated heights $H_f = \{h_0, h_1\}$, what is the equation for h(t) in the interval $[t_0, t_1]$ using linear interpolation?

d) Suppose we wish to achieve a smoother animation by using Catmull-Rom interpolation. What would the parametric derivatives of the interpolated curve be at t=4 and t=5?

e) Give 2D cubic Bezier control points for a Hermite interpolation given two sequential points (t_0, h_0) and (t_1, h_1) with associated derivatives (v_0, v_1).

13. Suppose we wish to rasterize two triangles T1 and T2 given screen space coordinates for their vertices as indicated in the diagram below.



a) If both triangles are black on a white background, a naive approach would be to fill in each pixel that contains any piece of either triangle. What kind of aliasing effect that would result?

b) Suppose T1 has RGB colour (16,16,16), T2 has RGB colour (0,0,16) and the background has RGB colour (0,0,0). If the rasterization was antialiased using the obvious approach called “unweighted area averaging”, what colour should pixel (3,5) be shaded? What colour should pixel (11,6) be shaded?

c) Analytic antialiasing is difficult. Explain an efficient approach that is commonly used in Ray-tracing and GPU accelerations.

14. Shadow maps are one method of generated shadows using OpenGL. Describe how to implement shadow maps

15. Bounding boxes are one method of increasing the speed of a ray tracer. Typically, the bounding boxes are aligned with the coordinate axes.
a) Discuss the benefits/costs of axis aligned bounding boxes vs bounding boxes that are not axis aligned.

b) Discuss the benefits/costs of using bounding spheres instead of bounding boxes.

16. In this question, we are working in 2 dimensions. All transformations map from the standard Cartesian frame to the standard Cartesian frame.
Let R(θ) be the matrix for a rotation about the origin of θ in the counter-clockwise direction.
Let T (v) be the matrix for a translation by v.
Let S(sx, sy) be the matrix for a non-uniform scale about the origin by an amount sx in the x direction and sy in the y direction.
Given a point p, two perpendicular unit vectors v and w, and two scale factors a and b, suppose we want to perform a non-uniform scale about p by a in direction v and b in direction w. Give the appropriate matrix product to achieve this transformation using the above notation for the matrices.
Note: You should not give expanded forms of the matrices; instead, your matrix product should be products of R(), T (), and S() (each operation may be used more than once). Also, these should be treated as matrices and not transformations (which is important for the order). Further assume that points and vectors are represented as column matrices.

17. Explain the term mathematical continuity (C_n) when joining two curves.

Consider the joint between two cubic Bezier curves. State and prove constraints on their control points to ensure:
(i) C0 continuity at the joint.

(ii) C1 continuity at the joint.

18. Given two matrices cow2wld and wld2cam, a cow can be transformed to the camera space using wld2cam * cow2wld.
Here, cow2wld which transfrom a cow from the object coordinates to the world coordinates, wld2cam which transform a cow from the world coordinates to the camera coordinates, Rx is the rotation matrix. You can use a.R, a.T to denote the rotation and translation part of the matrix, that is, a=a.T * a.R

a) describe how to rotate a cow along X-axis in the object space

b) describe how to rotate the cow along the X-axis of the world space.

c) describe how to rotate the cow along the X-axis of the camera space.