Student \#:
Name:

Write down answers in-between questions. Please answer using short sentences. The given spaces should be more than enough.

1. If the following 1D discrete filters are used to define 2D filters and applied to images, which filter goes with which operation? hints: assuming $a[i, j]=a_{1}[i] a_{1}[j]$, a filtering operation can be defined using the convolution operator * as follows:
$(a * b)[i, j]=\sum_{i^{\prime}, j^{\prime}} a\left[i^{\prime}, j^{\prime}\right] b\left[i-i^{\prime}, j-j^{\prime}\right]=\sum_{i^{\prime}} a_{1}\left[i^{\prime}\right]\left(\sum_{j^{\prime}} a_{1}\left[i^{\prime}\right] b\left[i-i^{\prime}, j-j^{\prime}\right]\right)$, where 1D
filter $a_{1}$ is one of the followings.
2. 


(a) sharpen
(b) blur
(c) differentiate (in some way)
(d) shift right and up
4.

3.

2. Biefly explain what is moire' pattern, what causes it, and how to prevent it.
3. (a) Briefly dscribe the problems of magnification and minification in texture-mapping.
(b) Briefly describe algorithms to reduce the artifacts caused by the minification (two algorithms) and magnification (one algorithm) of textures.

4 If we use each of the following 1D reconstruction filters to reconstruct a continuous function $g(x)$ from a sequence of samples $f[i]$ using continuous-discontinuous convolution, for which filters will $g(x)$ be $C^{0}$ ?
For which filters will $g(x)$ be $C^{1}$ ?
For which filters will $g(x)$ interpolate $f[i]$ ?
hints: the reconstructed function $g(x)$ is defined as $g(x)=\sum_{i} f[i] a(x-i)$ for a arbitrary sequence of samples $f[i]$ when each of the followings is $a(x)$.
1.

2.


3.

4.

5 Create a hierarchical model of the object in figure composed of four cylinders connected at the end. You have available the geometric primitive DrawCylinder(float $r$, float $h$ ), which draws a cylinder of radius $r$ and height $h$, whose center is the origin and where the height is along the x axis. Write a pseudo code that draws Figure 2 by moving from the position in Figure 1 to the position in Figure 2. (Hint: use a chain of matrix multiplications)


Figure 1.


The other functions you may need are:

```
glRotate(angle, axisx, axisy, axisz)
glTranslate(dx,dy,dz)
glPushMatrix()
glPopMatrix()
```

6. Consider the second degree polynomial $p(u)=c_{0}+c_{1} u+c_{2} u^{2}=\boldsymbol{u}^{T} \boldsymbol{c}=\left[1 u u^{2}\right]\left[c_{0} c_{1} c_{2}\right]^{T}$ and the control point $\boldsymbol{p}=\left[p_{0} p_{1} p_{2}\right]^{T}$. Derive the matrix A so that the coefficients $\boldsymbol{c}=A^{-1} \boldsymbol{p}$ make $\mathrm{p}(\mathrm{u})$ interpolate all control points (for $u \in[0,1], u_{0}=0, u_{1}=0.5, u_{2}=1$ ).
7. Bezier polynomials can be rendered efficiently with recursive subdivision. It is common to convert a non-Bezier polynomial to an equivalent Bezier polynomial in order to use these rendering techniques. Describe how to do this mathmatically.
(b) conversion to Beziers
(a) recursive subdivision.
8. Describe one method to manage the speed-realism tradeoff using level of detail (LOD).
9. Briefly describe the Phong shading method discussing how it integrates in the modern graphics pipeline, its advantages and disadvantages.
10. The tree below represents a quadtree subdivision of a square. The left most branch is the upper left quarter, the next branch is the upper right, the third branch is the lower left, and the right most branch is the bottom left of a region. The leaf nodes are labeled A-J.
Subdivide the square below to indicate the subdivision of the quad tree. Label each portion of the square with the appropriate label from the tree.

11. Below are four curves and their "control points/polygon." Two of the control polygons are the Bezier control polygon for the curve drawn with it; the other four are not. Indicate which of the control polygons are Bezier control polygons for the corresponding curve and which are not. Justify your answer for the control polygons that are not Bezier control polygons. You may assume that none of the control points overlap or are repeated.
a.

b.

C.

d.

12. Describe the algorithm used to determine whether a ray intersects a polygon.
13. Suppose we have an image of a gray elephant, with an alpha matte to delineate foreground from background. The image is stored with non-premultiplied alpha (unusually).

a. If we accidentally use the image in a program that expects premultiplied alpha but without premultiplying the alpha values, will the partially transparent edges come out too dark, about right, or too light if the background is:
(a) solid black
(b) solid white
(c) about the same color as the elephant

Explain why by writing down the equations for the correct and incorrect results.
Hints: when using premultiplied alpha $\begin{gathered}\mathrm{C}=\mathrm{A} \text { over } \mathrm{B} \text { becomes } \\ c_{C}^{\prime}=c_{A}^{\prime}+\left(1-\alpha_{A}\right) c_{B}^{\prime}\end{gathered}$
14. (a) Construct a summed area table from the texture below, and explain how you can calculate the average value of the middle 4 pixels of the texture using the summed area table.

(b) How is this related to texture filtering?
15. Write down a sequence of numbers in the order that the k -d tree is traversed when checking intersections between a ray and the scene. Each number at an internal node represents the intersection test between the ray and the axis-aligned bounding box corresponding to the node. The numbers at the leaf nodes represents the intersection test between the ray and the primitives. The number at the arrow represents the tree traversal. Omit numbers that correspond to pruned operations (that are not executed). Do not exclude intersection tests that fails; for example, the correct answer is " 1 " if the ray doesn't collide with the outer-most bounding box, not an empty string """.


Ray 1:

Ray 2:

