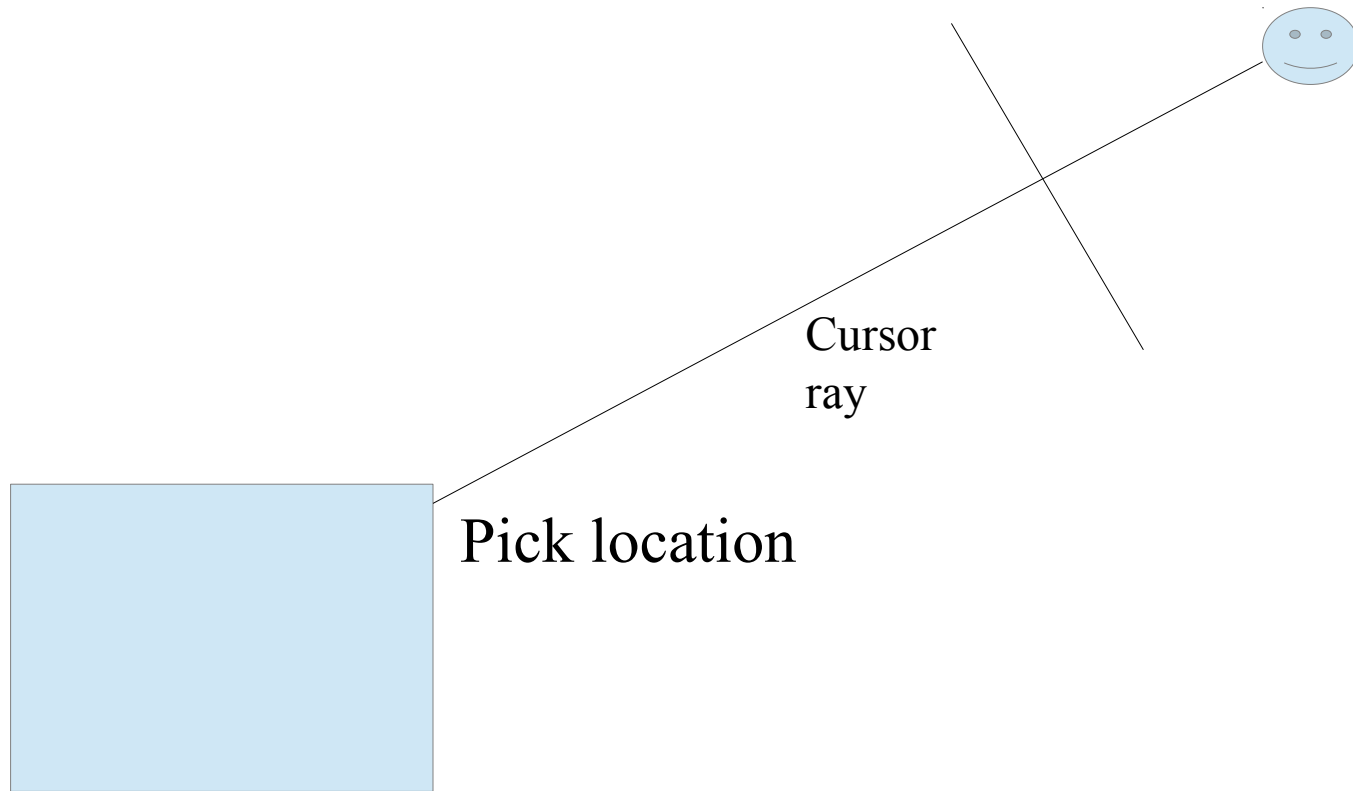


Collision Detection

(c) Steve Rotenberg, UCSD

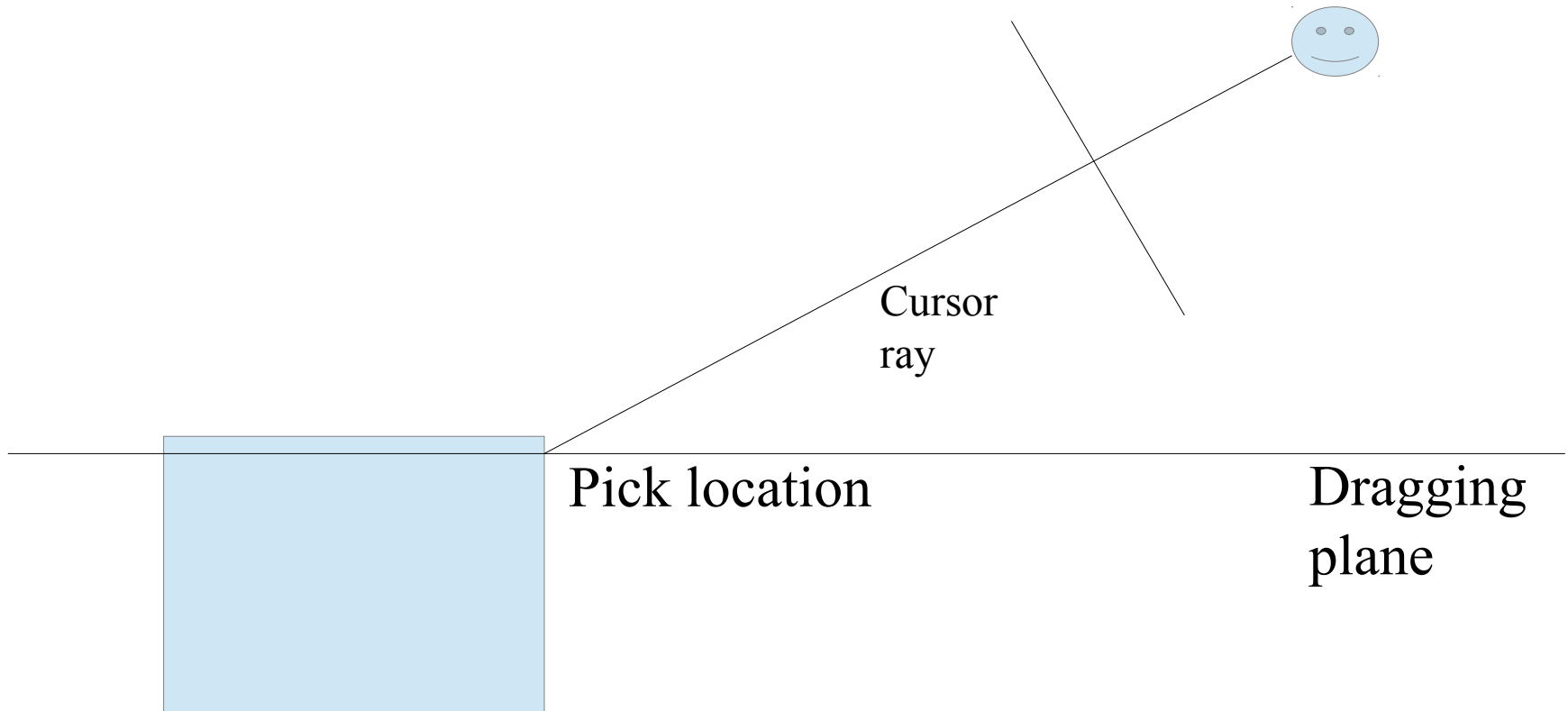
HW2

- How to implement precise dragging



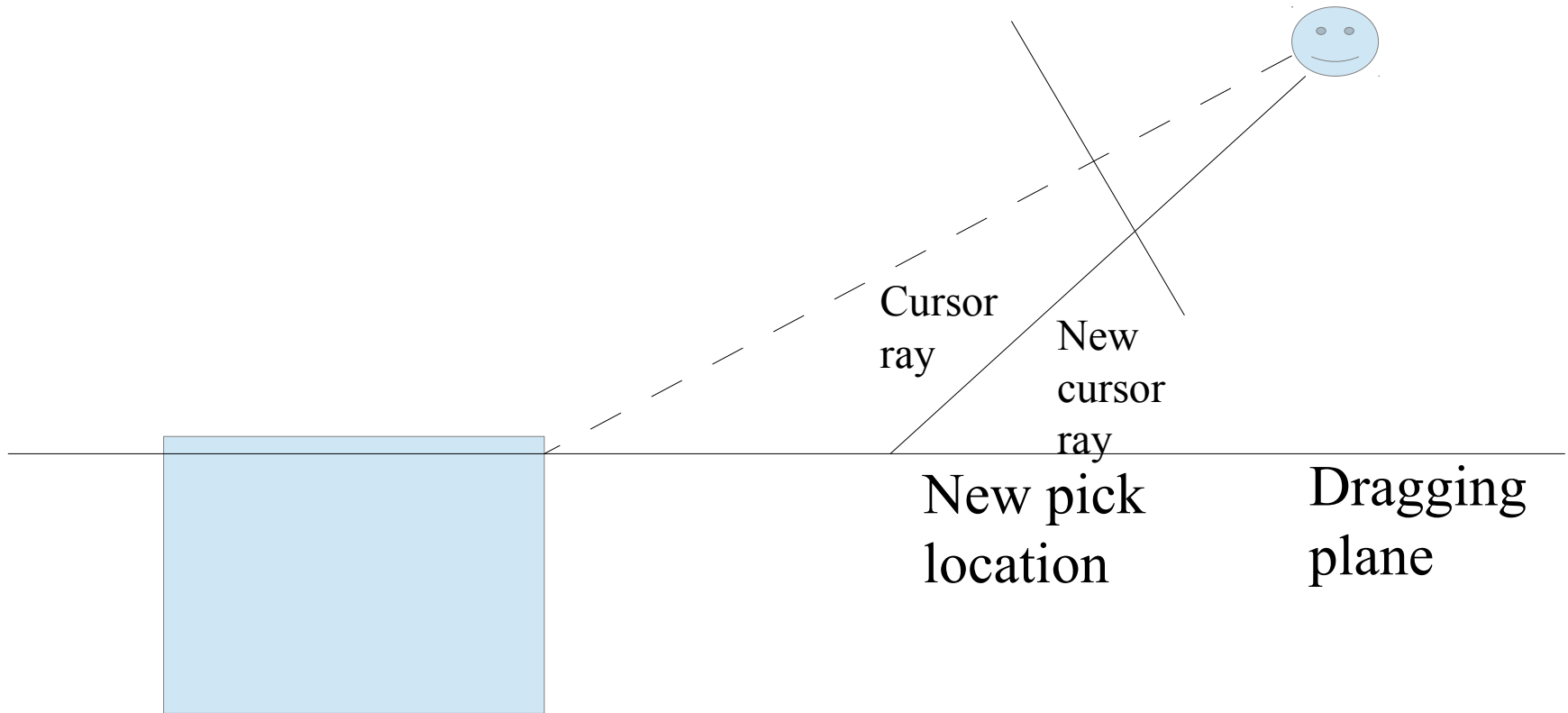
HW2

- How to implement precise dragging



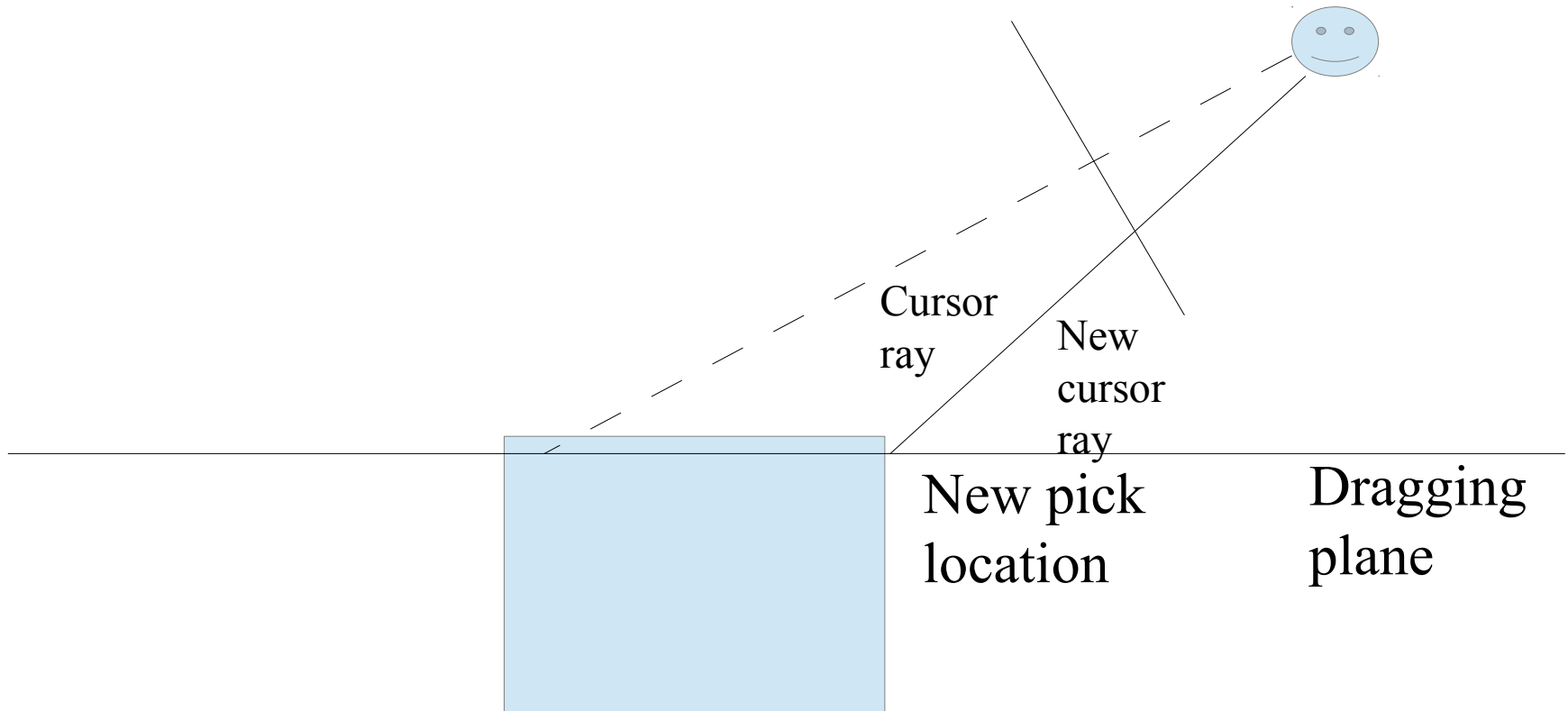
HW2

- How to implement precise dragging



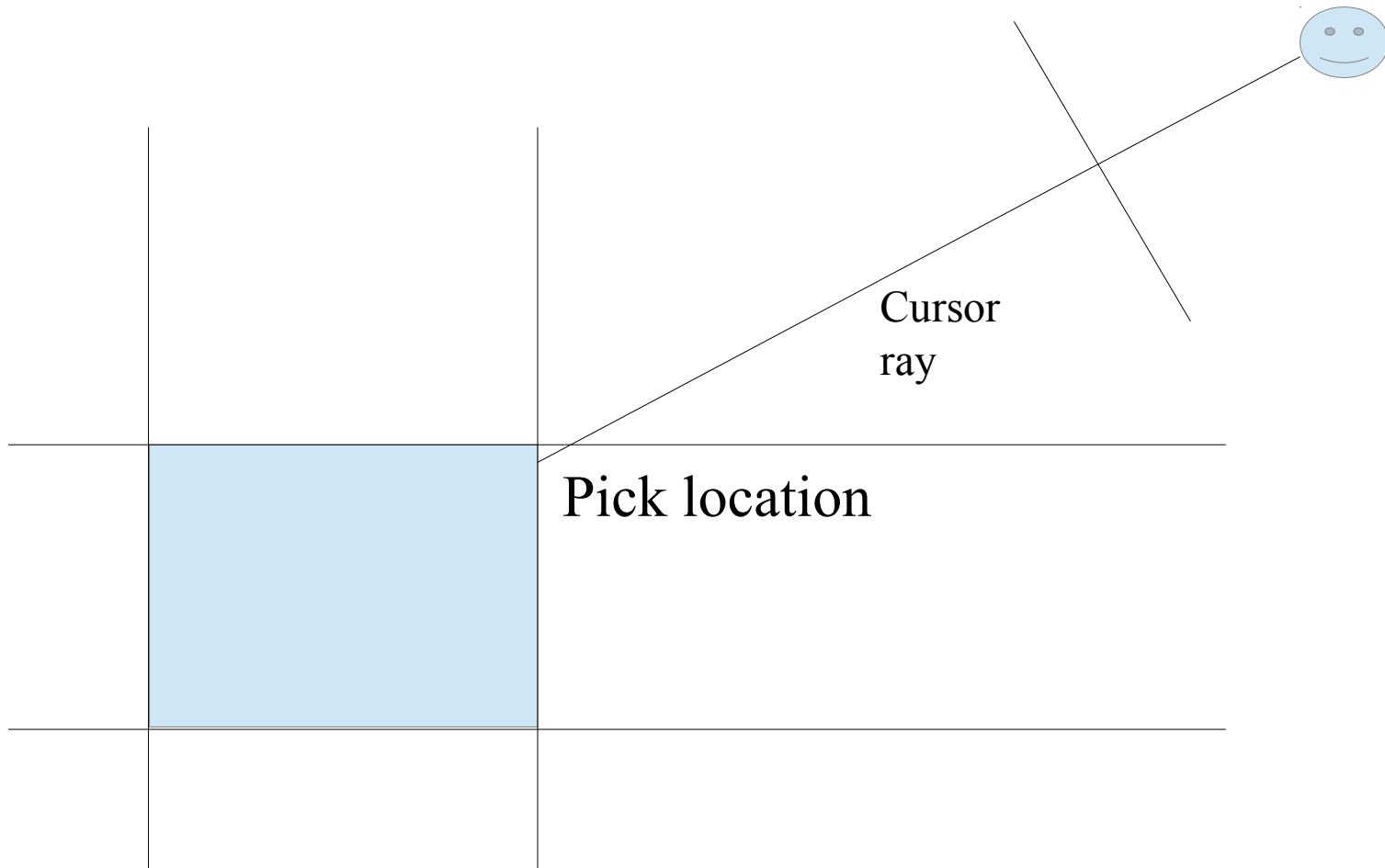
HW2

- How to implement precise dragging

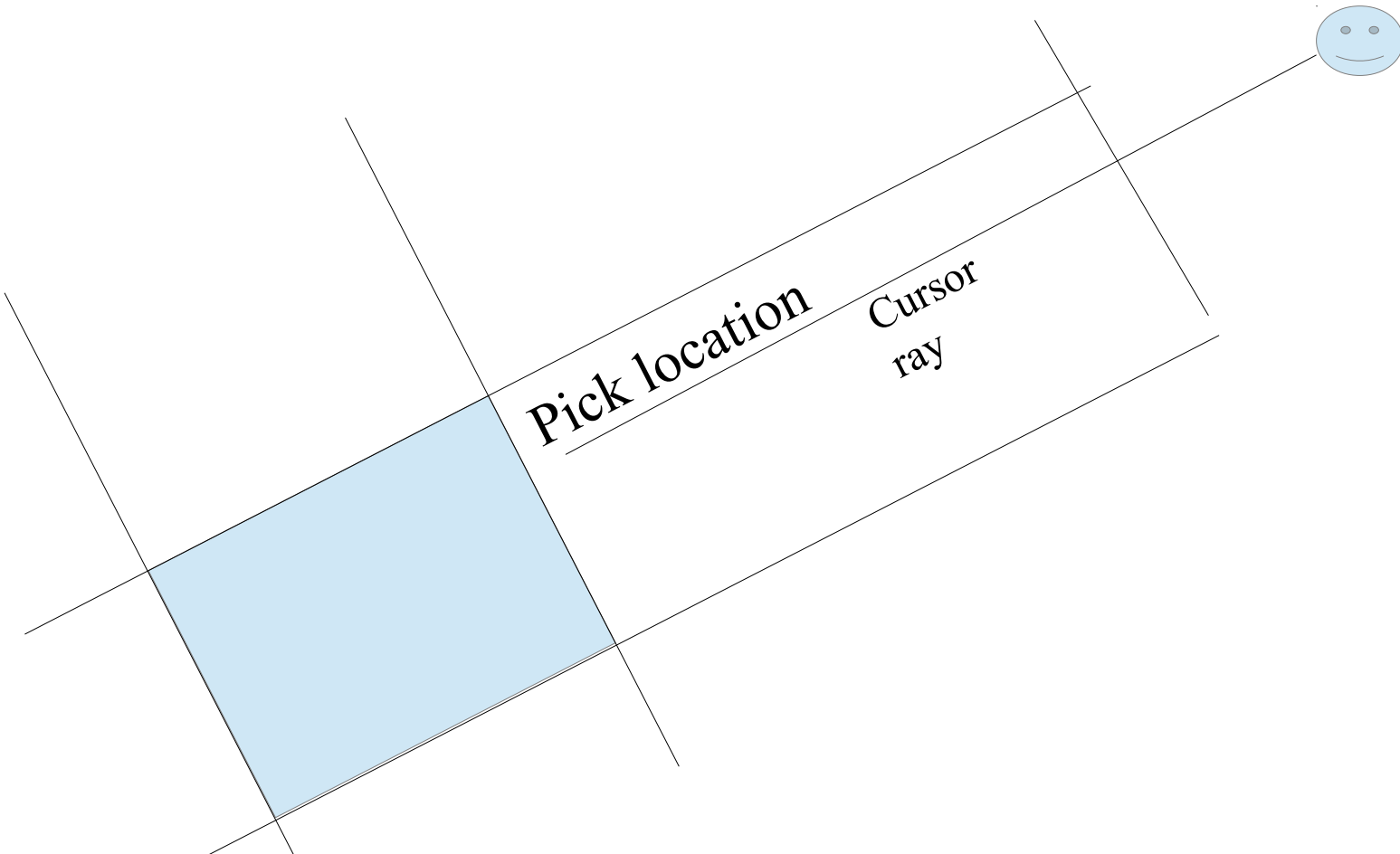


More information on the hanyang portal

How to compute pick location

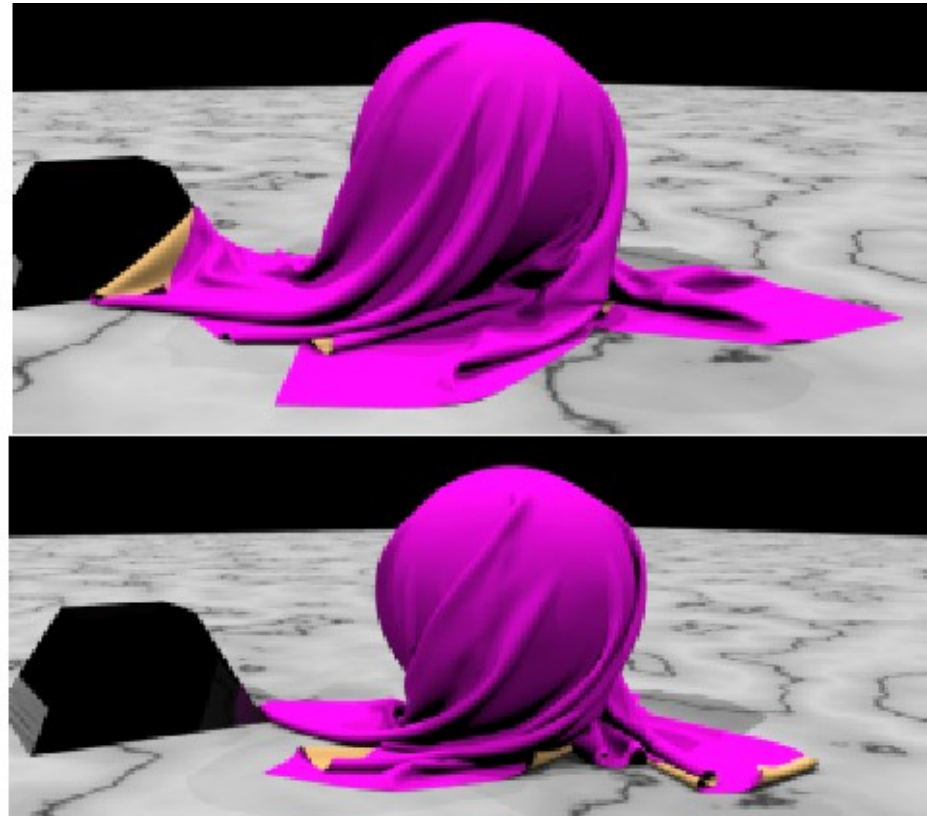


Consider this case!



Collision Detection & Response

- Rigid-object colliding with another rigid object is relatively easy
- Cloth colliding with rigid objects is tricky
- Cloth colliding with itself is even trickier

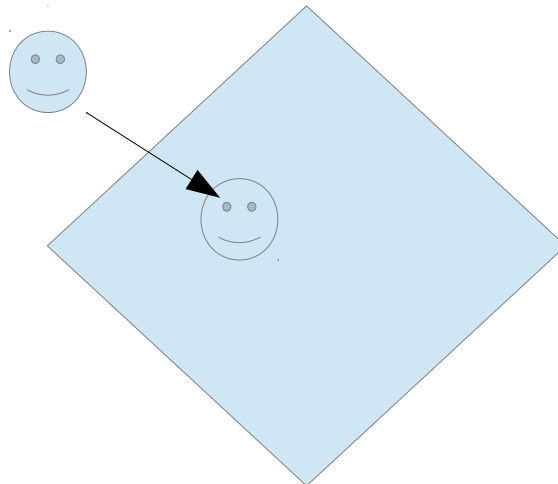


Collision Detection

- Applications
 - Physics simulation
 - Ray-tracing
 - User-interactions
 - Games (Path-planning, AI, etc ...)
- Main subjects
 - Intersection testing (triangles, spheres, lines...)
 - Optimization structures (octree, BSP...)
 - Pair reduction (reducing N^2 object pair testing)

Intersection Testing

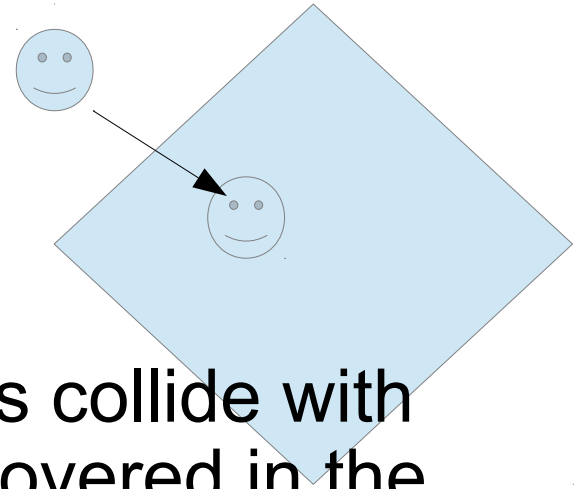
- General goals: given two objects with current and previous orientations specified, determine if, where, and when the two objects intersect
- Alternative: given two objects with only current orientations, determine if they intersect



Primitives

- We often deal with various different ‘primitives’ that we describe our geometry with. Objects are constructed from these primitives
- Examples
 - Triangles
 - Spheres
 - Cylinders
 - AABB = axis aligned bounding box
 - OBB = oriented bounding box
- At the heart of the intersection testing are various primitive-primitive tests

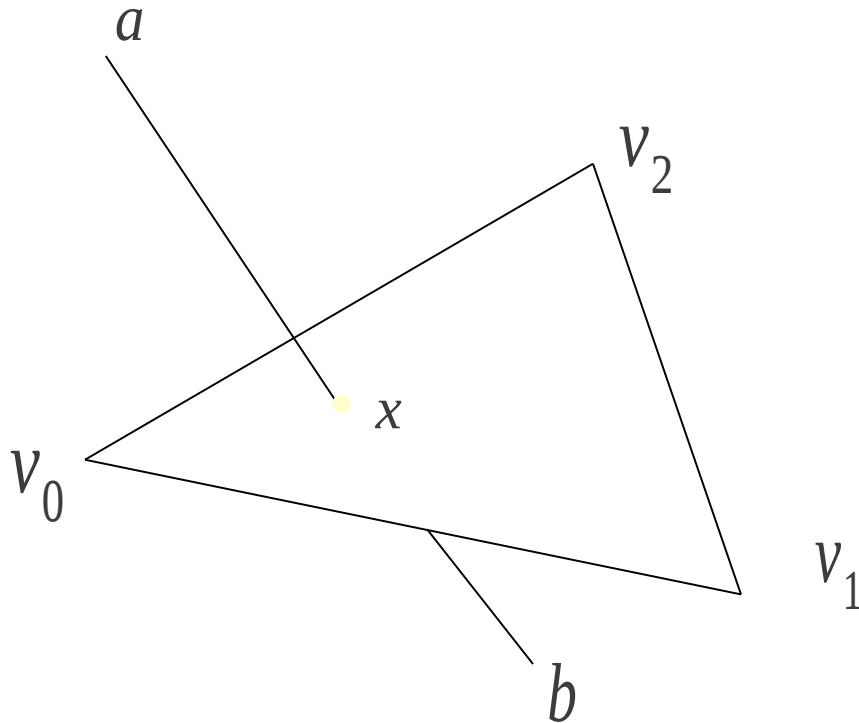
Particle Collisions



- The problem of testing if particles collide with solid objects had already been covered in the ray-tracing chapter.
 - A particle can be treated as a line segment from it's previous position to it's current position
- If a particle is colliding against static objects, then we just need to test if the line segment intersects the object
- Colliding against moving objects requires some additional modifications that we will look at

Segment vs. Triangle

- Does segment **ab** intersect triangle $\mathbf{v_0v_1v_2}$?

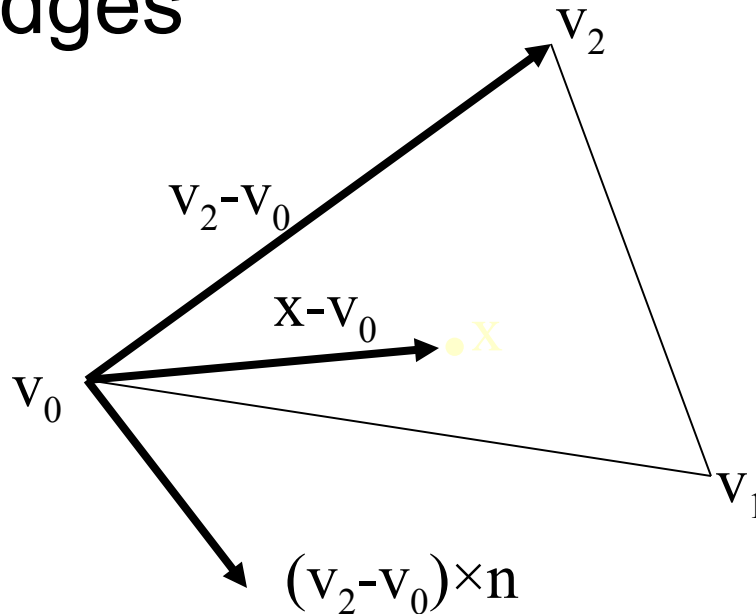


Segment vs. Triangle

- Is point x inside the triangle?

$$(x-v_0) \cdot ((v_2-v_0) \times n) > 0$$

- Test all 3 edges



Faster Way

- Reduce to 2D: remove smallest dimension
- Compute barycentric coordinates

$$\mathbf{x}' = \mathbf{x} - \mathbf{v}_0$$

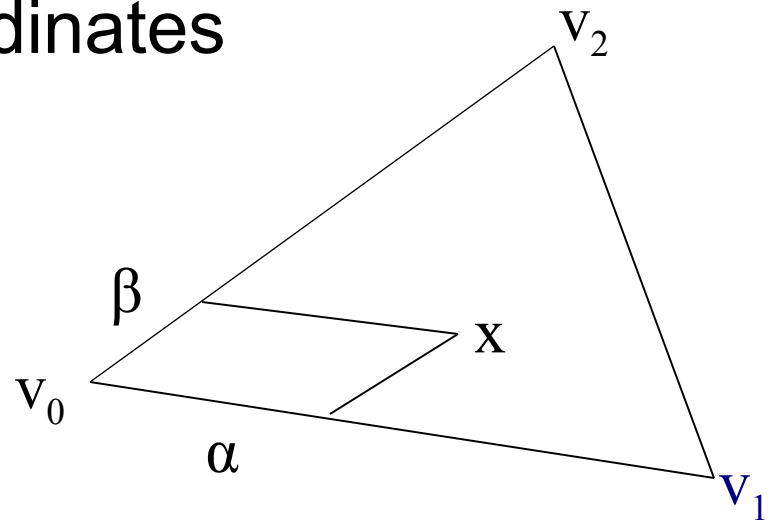
$$\mathbf{e}_1 = \mathbf{v}_1 - \mathbf{v}_0$$

$$\mathbf{e}_2 = \mathbf{v}_2 - \mathbf{v}_0$$

$$\alpha = (\mathbf{x}' \times \mathbf{e}_2) / (\mathbf{e}_1 \times \mathbf{e}_2)$$

$$\beta = (\mathbf{x}' \times \mathbf{e}_1) / (\mathbf{e}_1 \times \mathbf{e}_2)$$

- Reject if $\alpha < 0$, $\beta < 0$ or $\alpha + \beta > 1$



Segment vs. Mesh

- To test a line segment against a mesh of triangles, simply test the segment against each triangle
- Often, we are interested in only the ‘first’ hit along the segment from **a** to **b**.
- Testing against lots of triangles in a large mesh can be time consuming. We will look at ways to optimize this later

Segment vs. Moving Mesh

- M_0 is the object's matrix at time t_0
- M_1 is the matrix at time t_1
- Compute delta matrix:

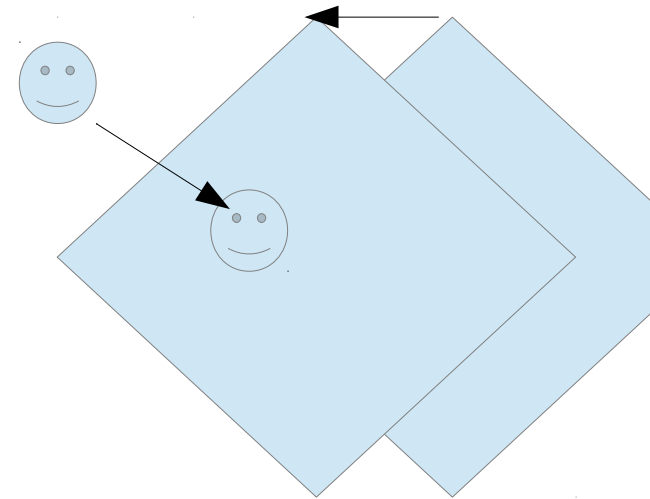
$$M_1 = M_0 \cdot M_\Delta$$

$$M_\Delta = M_0^{-1} \cdot M_1$$

- Transform a by M_Δ

$$a' = a \cdot M_\Delta$$

- Test segment a'b against object with matrix M_1



Segment vs. Moving Mesh

- M_0 is the object's matrix at time t_0
- M_1 is the matrix at time t_1
- Compute delta matrix:

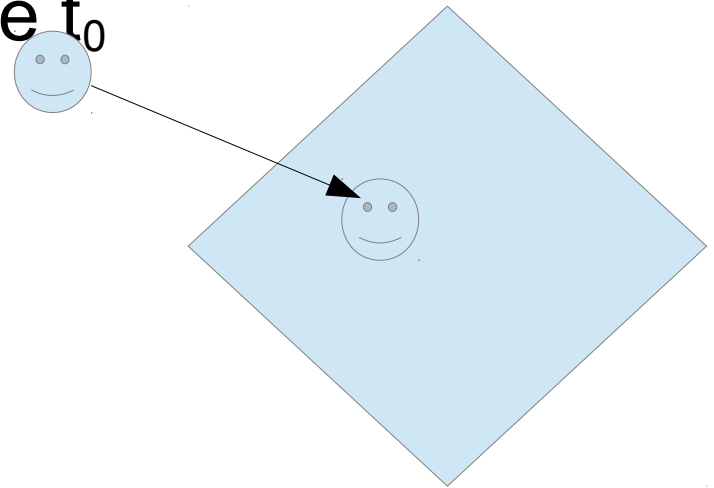
$$M_1 = M_0 \cdot M_\Delta$$

$$M_\Delta = M_0^{-1} \cdot M_1$$

- Transform a by M_Δ

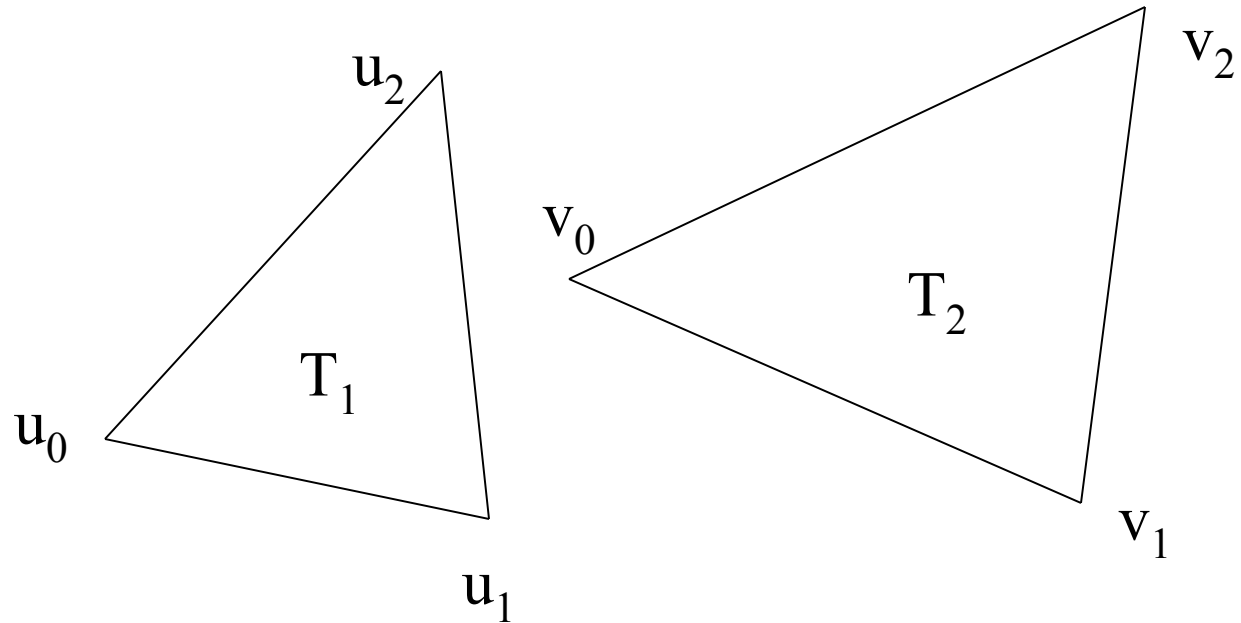
$$a' = a \cdot M_\Delta$$

- Test segment a'b against object with matrix M_1



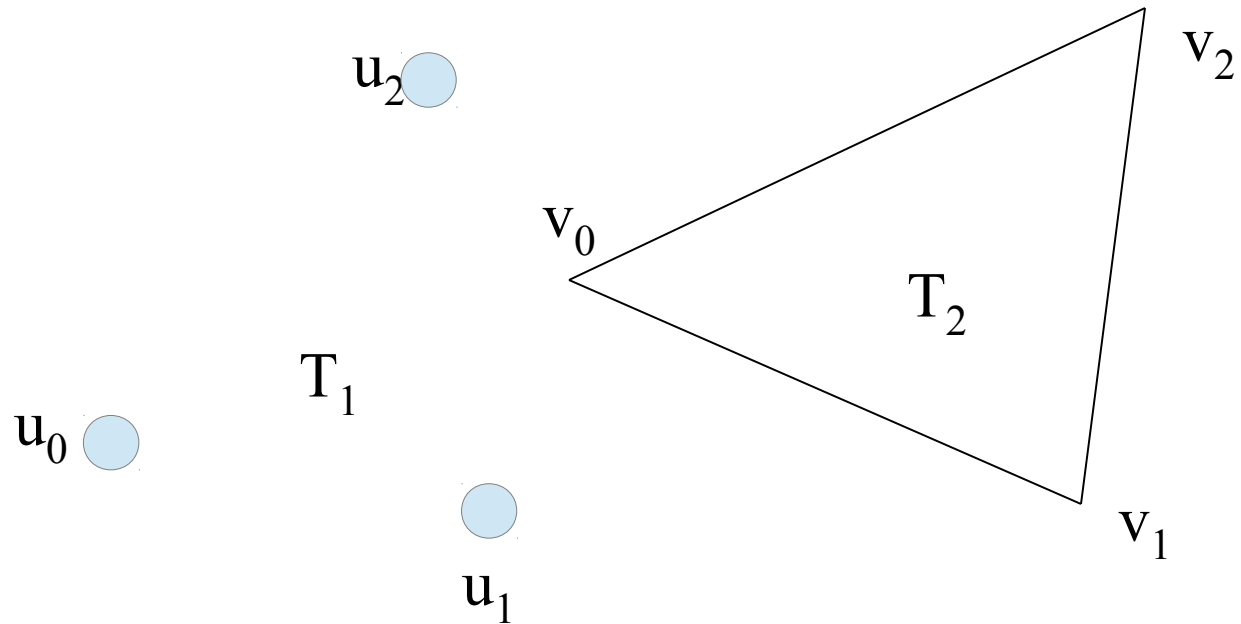
Triangle vs. Triangle

- Given two triangles: $T_1 (u_0u_1u_2)$ and $T_2 (v_0v_1v_2)$



1. Point-to-Face test

- Given two triangles: $T_1 (u_0u_1u_2)$ and $T_2 (v_0v_1v_2)$

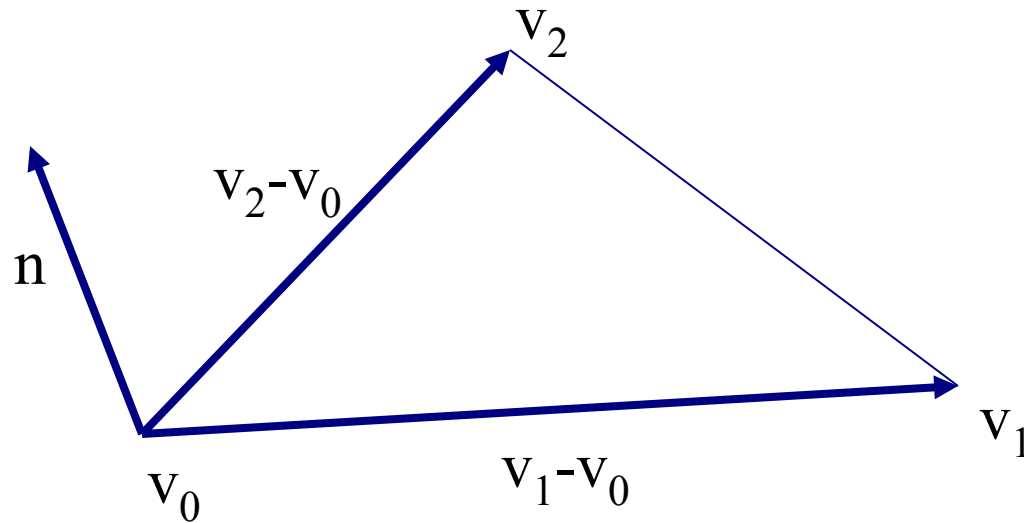


1. Point-to-Face test

Step 1: Compute plane equations

$$\mathbf{n}_2 = (\mathbf{v}_1 - \mathbf{v}_0) \times (\mathbf{v}_2 - \mathbf{v}_0)$$

$$d_2 = -\mathbf{n}_2 \cdot \mathbf{v}_0$$

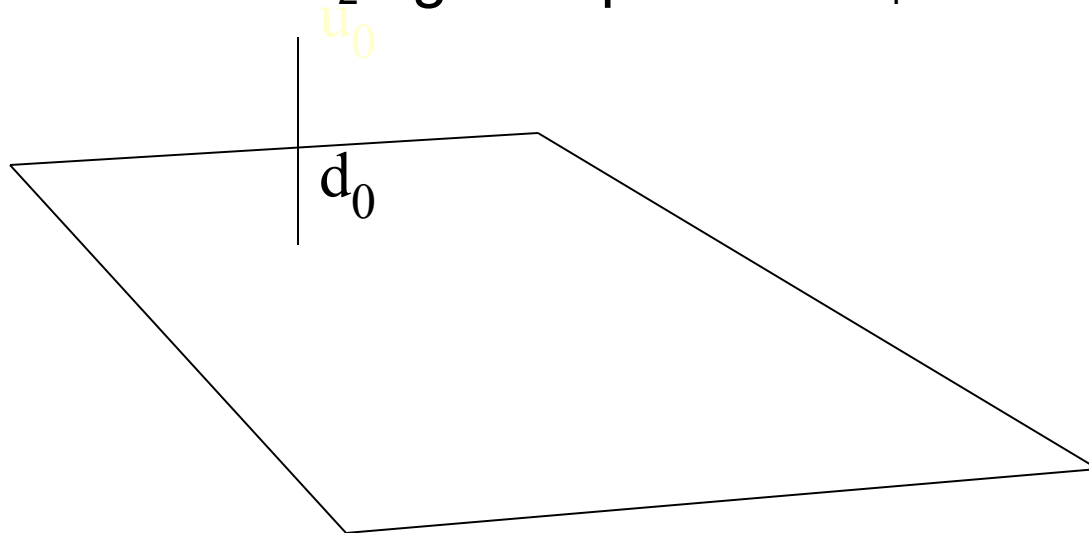


1. Point-to-Face test

- Step 2: Compute signed distances of T_1 vertices to plane of T_2 :

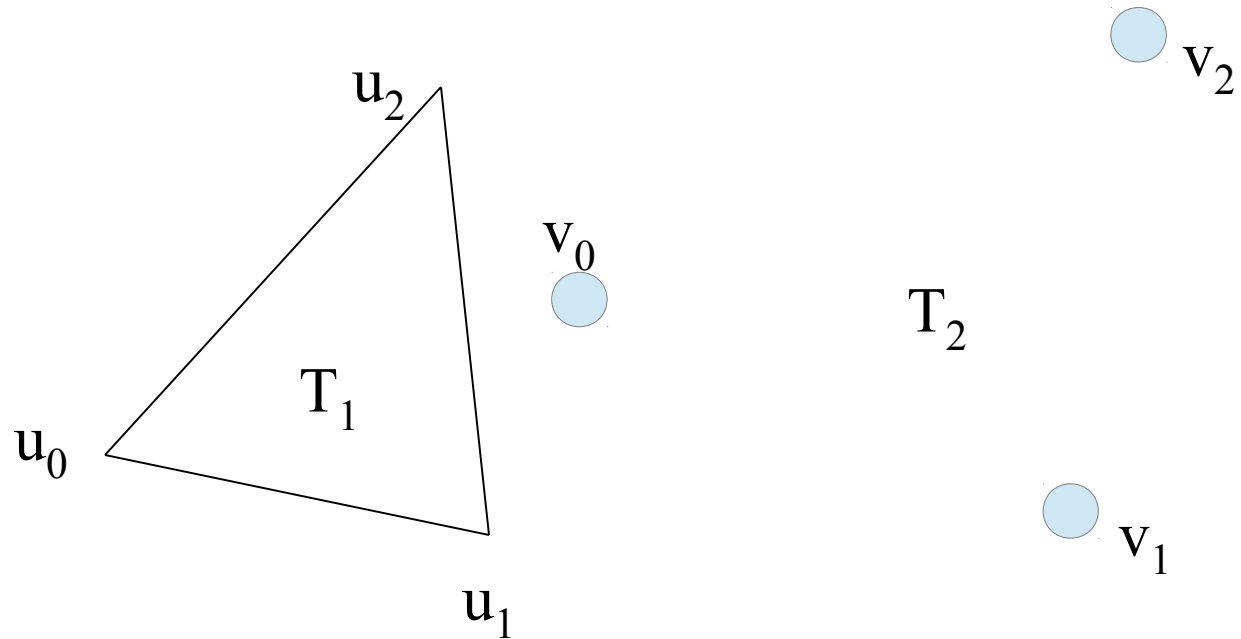
$$d_i = n_2 \cdot u_i + d_2 \quad (i=0,1,2)$$

- Reject if all $d_i < 0$ or all $d_i > 0$
- Repeat for vertices of T_2 against plane of T_1



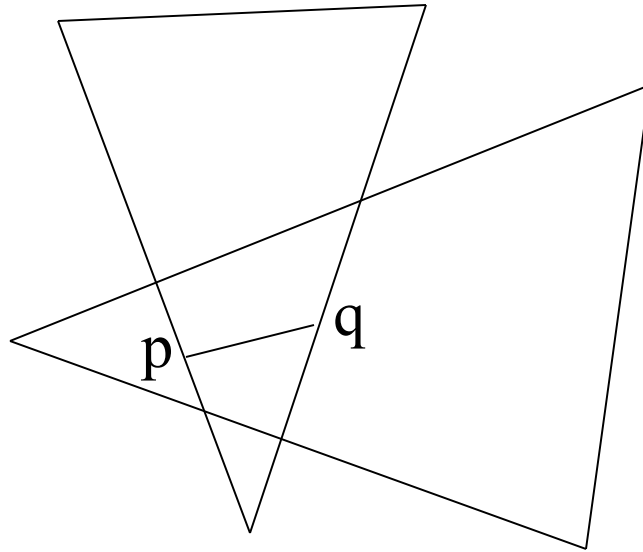
2. Face-to-Point test

- Given two triangles: $T_1 (u_0u_1u_2)$ and $T_2 (v_0v_1v_2)$



Triangle vs. Triangle

- Step 3: Find intersection points
- Step 4: Determine if segment pq is inside triangle or intersects triangle edge



Mesh vs. Mesh

- Geometry: points, edges, faces
- Collisions: p2p, p2e, p2f, e2e, e2f, f2f
- Relevant ones: p2f, e2e (point to face & edge to edge)
- Multiple simultaneous collisions

Moving Mesh vs. Moving Mesh

- Two options: 'point sample' and 'extrusion'
- Point sample:
 - If objects intersect at final positions, do a binary search backwards to find the time when they first hit and compute the intersection
 - This approach can tend to miss thin objects
- Extrusion:
 - Test '4-dimensional' extrusions of objects
 - In practice, this can be done using only 3D math

Moving Meshes: Extrusion

- Use 'delta matrix' trick to simplify problem so that one mesh is moving and one is static
- Test moving vertices against static faces (and the opposite, using the other delta matrix)
- Test moving edges against static edges (moving edges form a quad (two triangles))

Intersection Issues

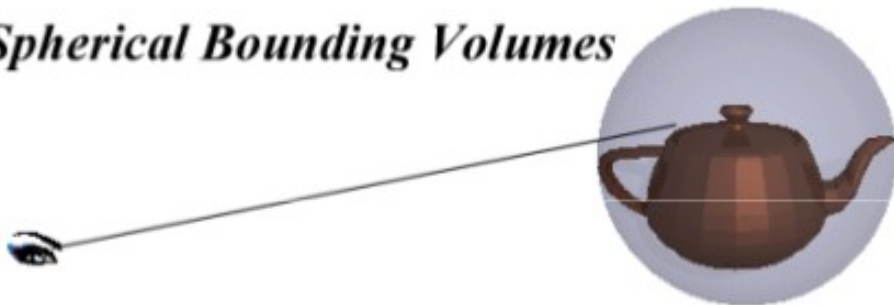
- Performance
- Memory
- Accuracy
- Floating point precision

Collision Optimization

Bounding volumes

- Enclose complex objects within a simple to intersect objects
 - If the ray does not intersect the simple object then its contents can be ignored
- The likelihood that it will strike the object depends on how tightly the volume surrounds the object.
 - Spheres are simple, but not tight
 - Axis-aligned bounding boxes often better

Spherical Bounding Volumes



Axis-Aligned Bounding Boxes

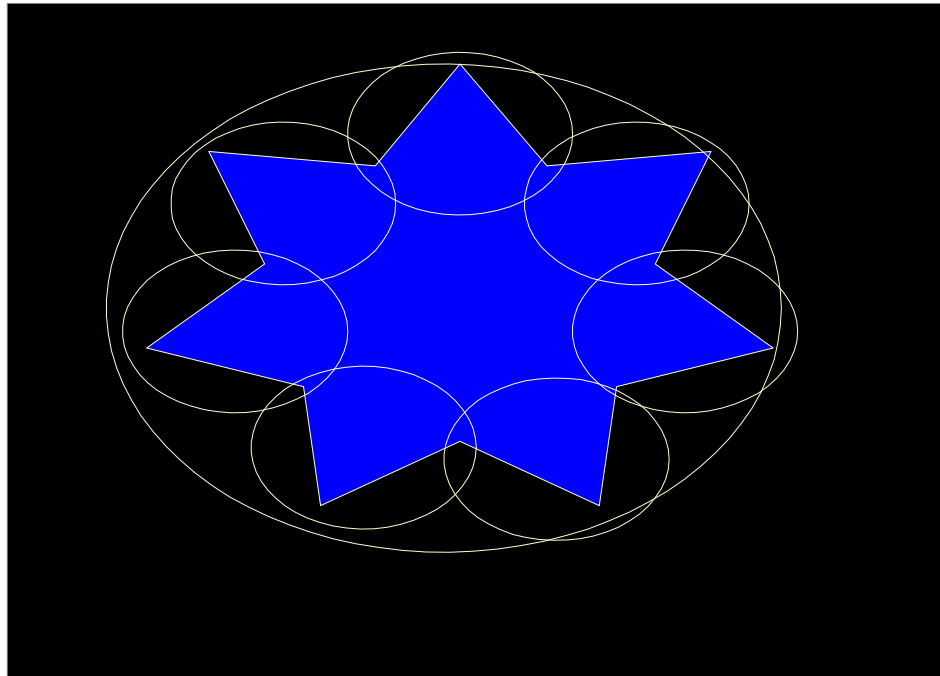


Optimization Structures

- BV, BVH (bounding volume hierarchies)
 - Octree
 - KD tree
 - BSP (binary separating planes)
 - OBB tree (oriented bounding boxes- a popular form of BVH)
 - K-dop tree
- Uniform grid
- Hashing
- Dimension reduction

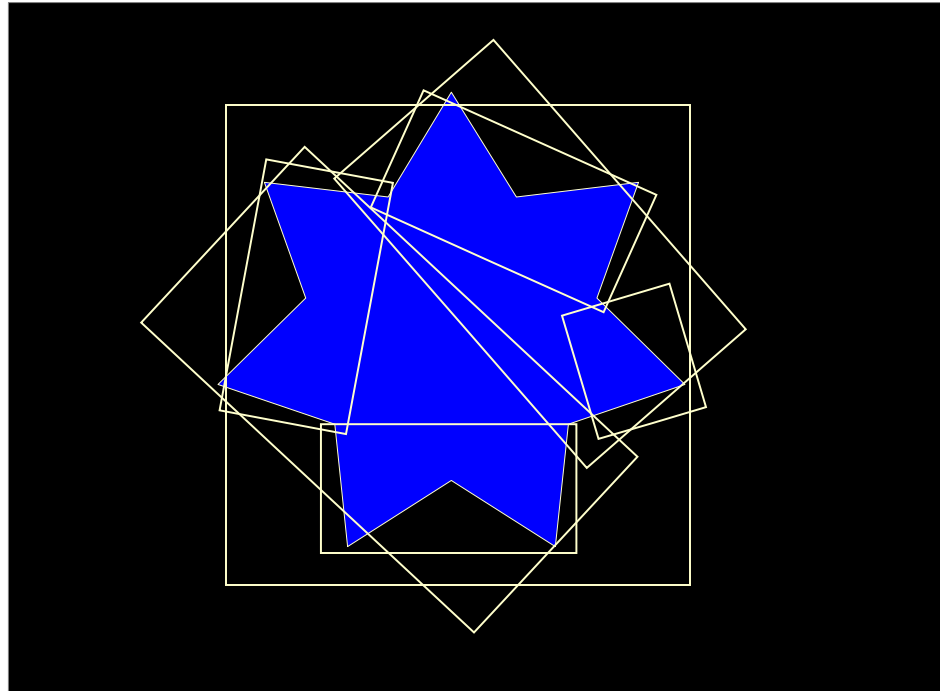
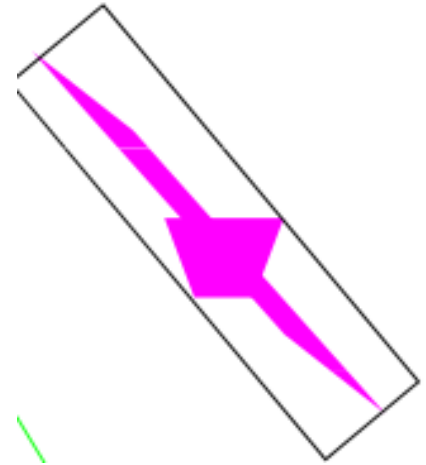
Sphere trees [Whitted80]

- Cheap to compute
- Cheap test
- Potentially very bad fit



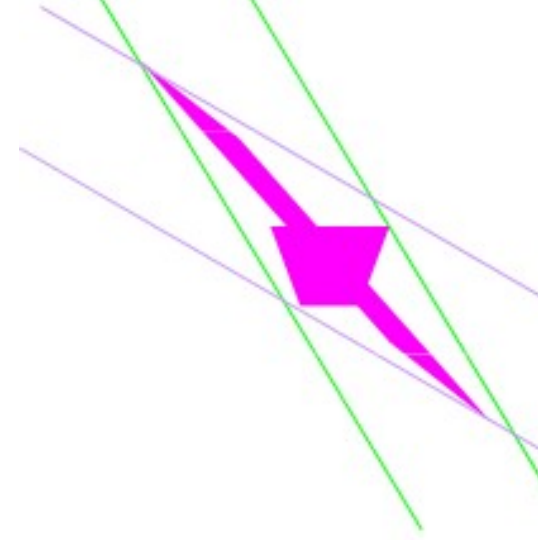
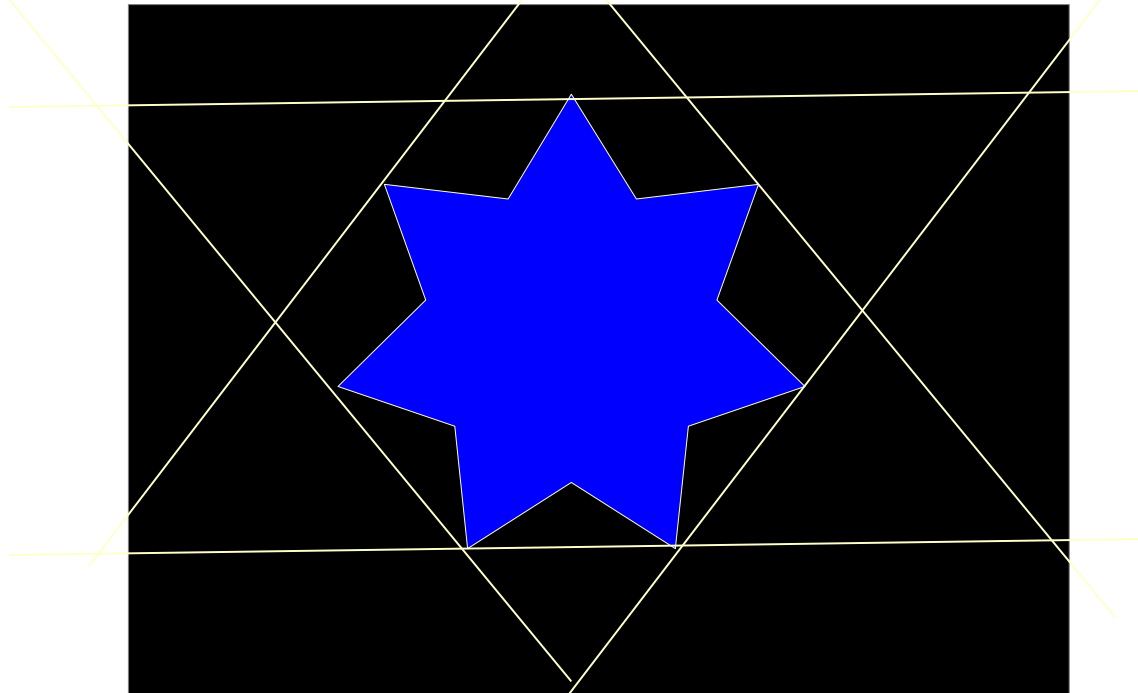
OBB Trees

- Oriented Bounding Box
 - Fairly cheap to compute
 - Fairly Cheap test
 - Generally fairly tight



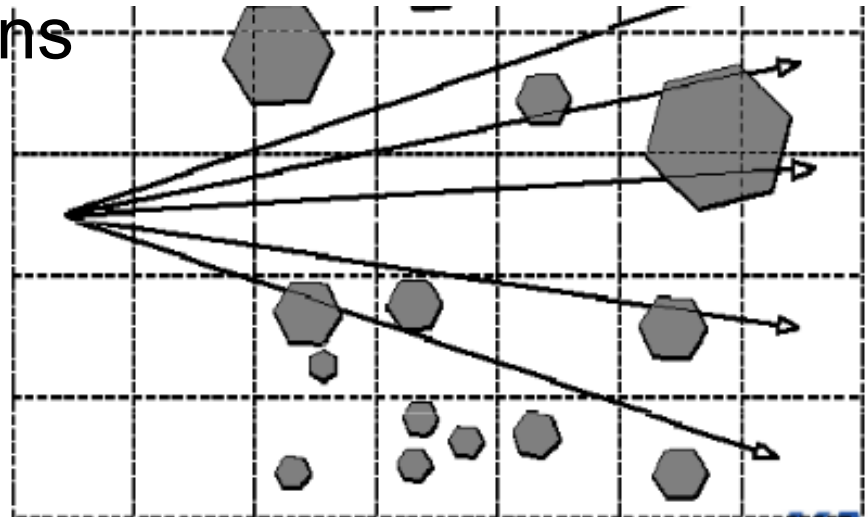
K-Dops

- More expensive to compute
- Fairly cheap test
- Can be tighter than OBB

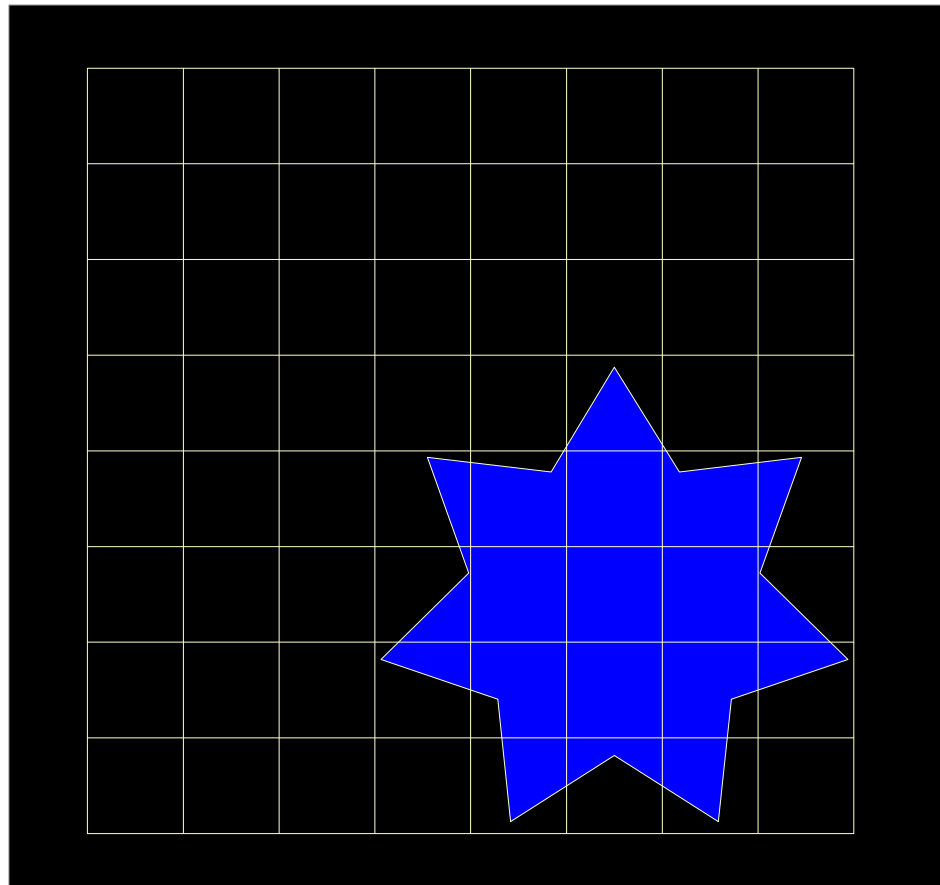


Spatial Subdivision

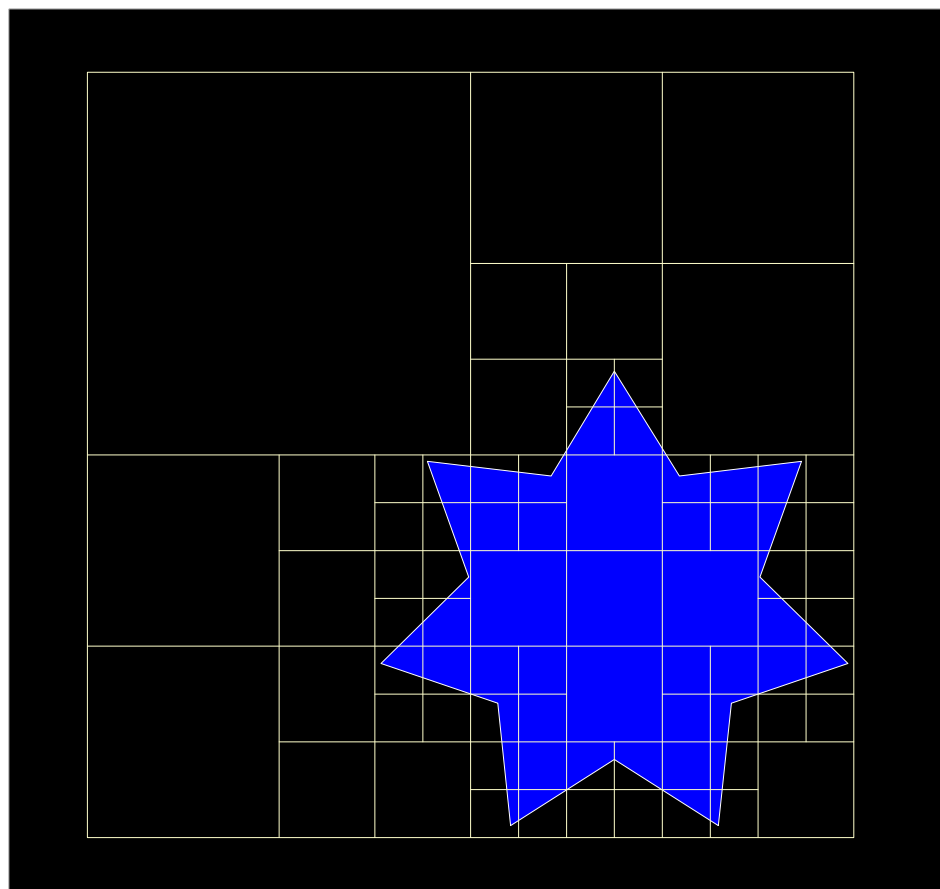
- Idea: Devide space into subregions
 - Place objects within a subregion into a list
 - Only traverse the lists of subregions that the ray passes through
- “Mailboxing” used to avoid multiple test with objects in multiple regions
- Many types
 - Uniform grid
 - Octree
 - BSP tree
 - kd-tree



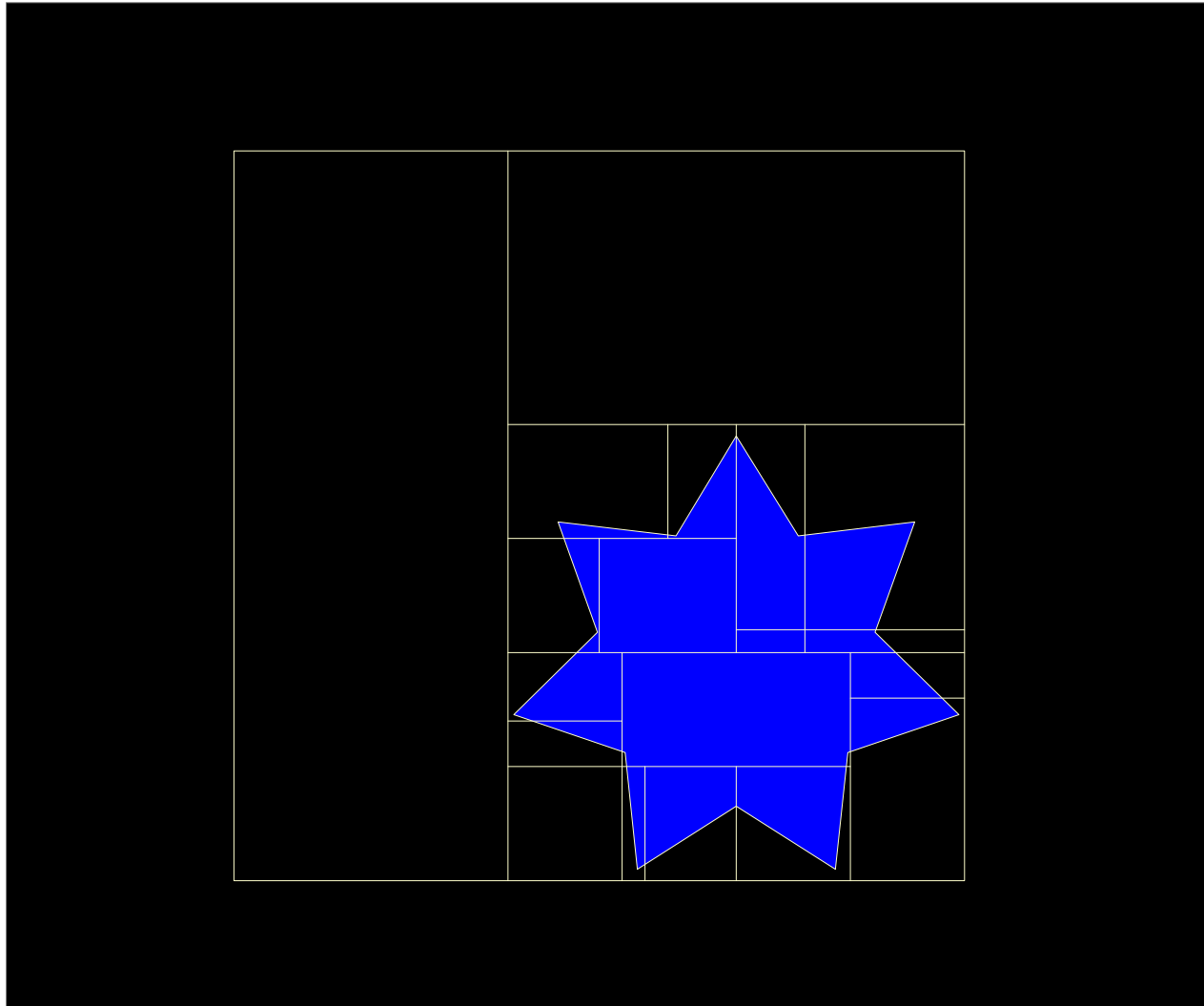
Uniform Grids



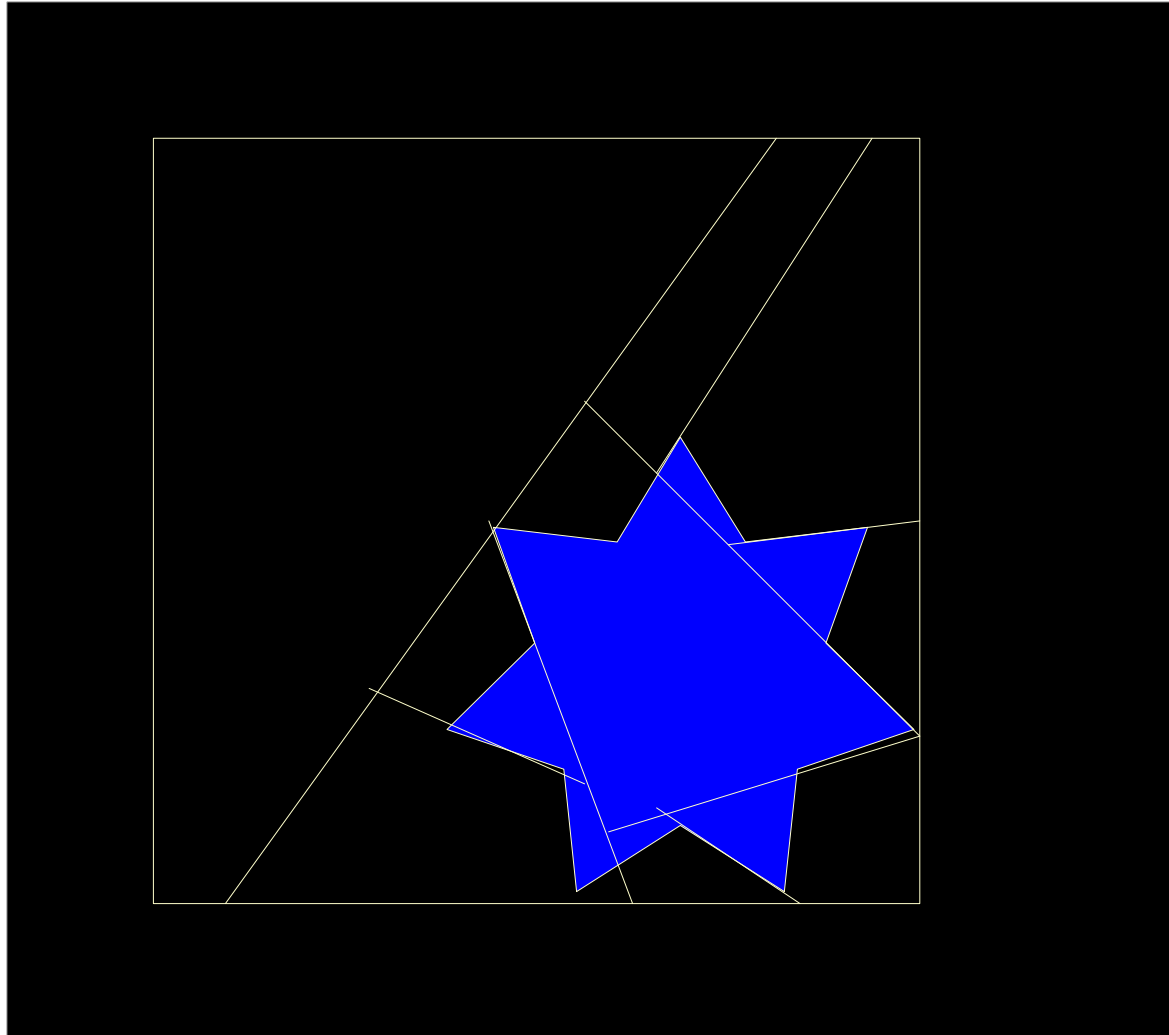
Octrees



KD Trees

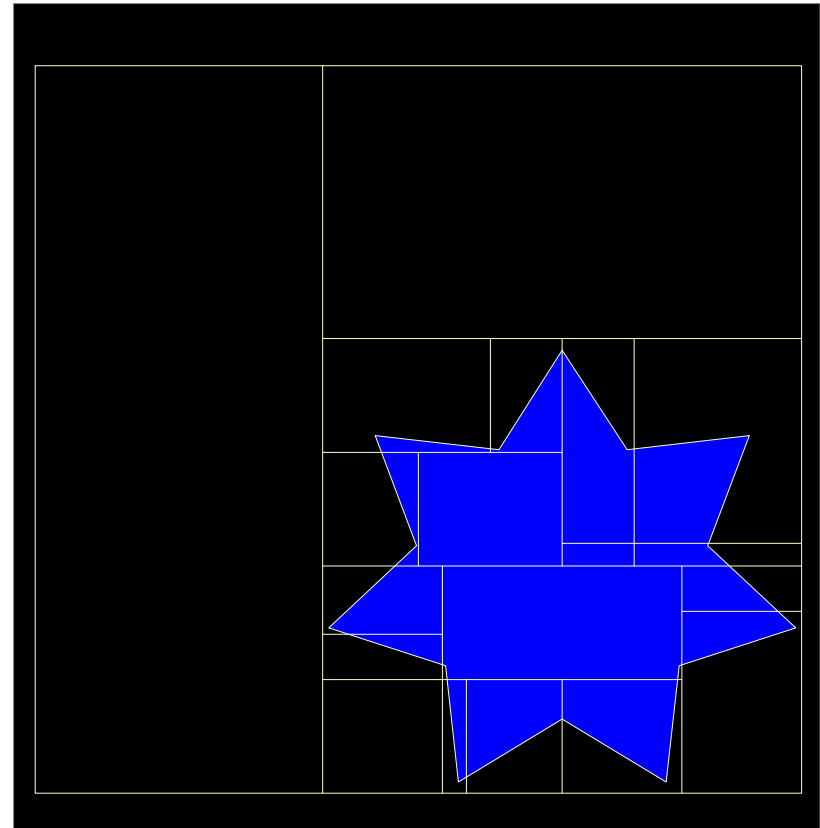


BSP Trees

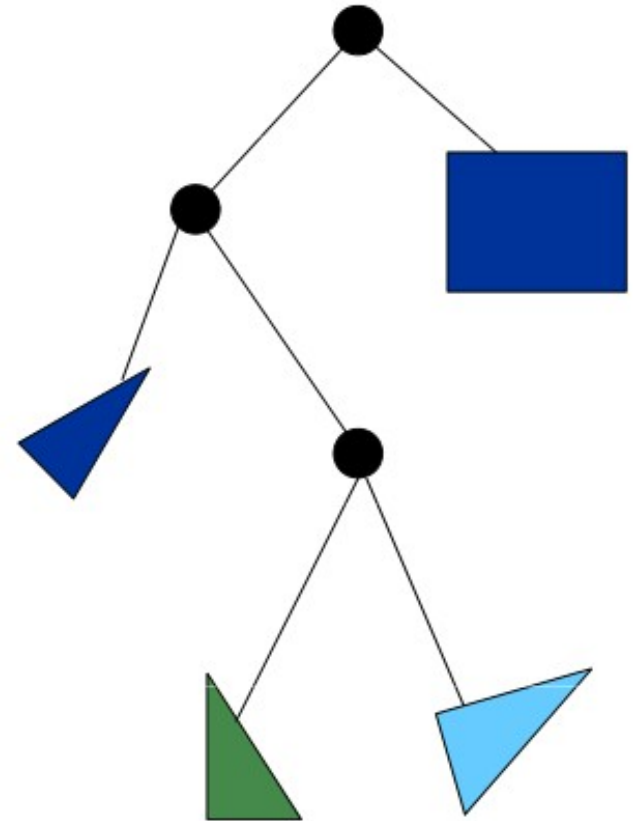
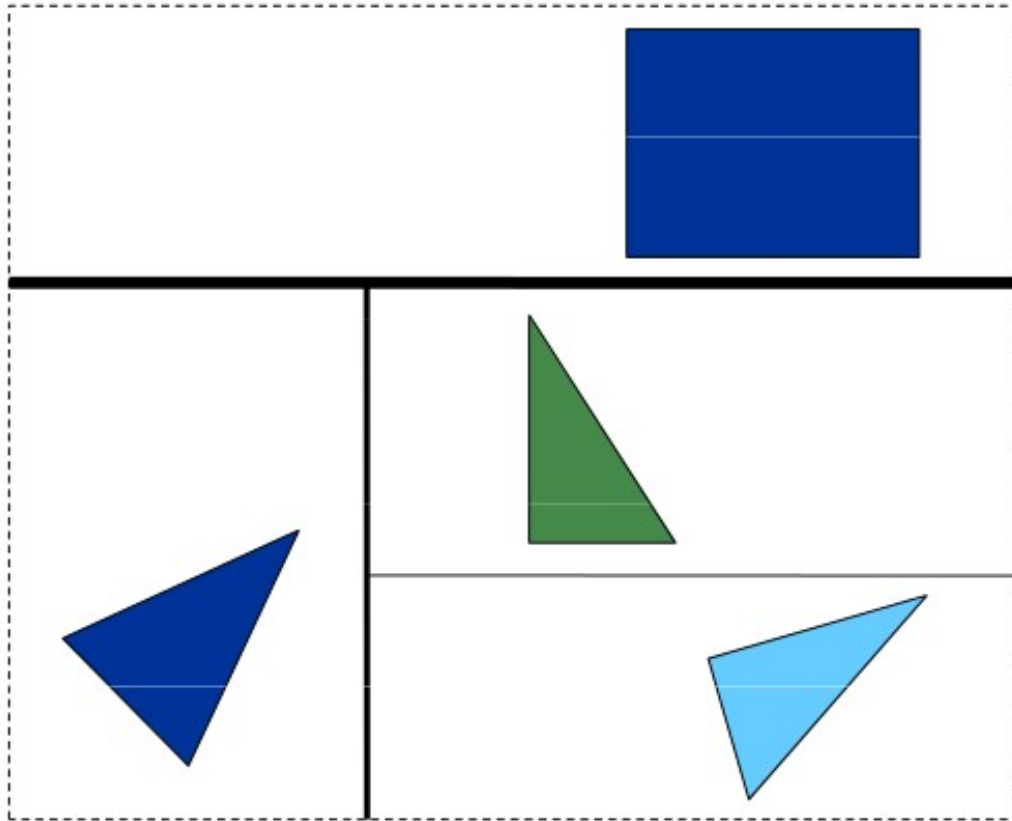


Overview of kd-Trees

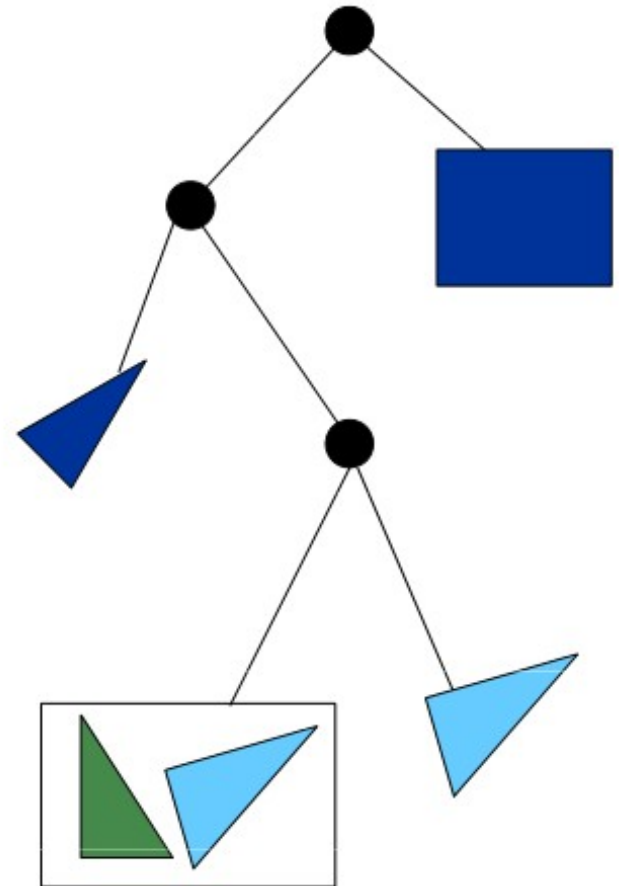
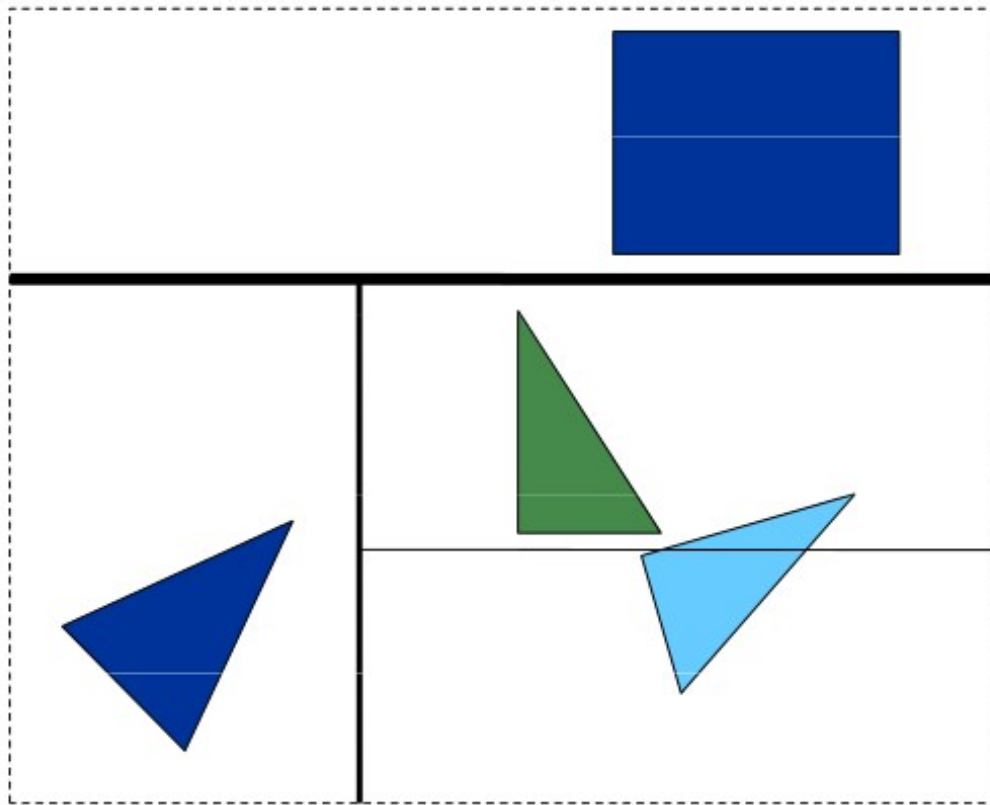
- Binary spatial subdivision
- (special case of BSP tree)
 - Split planes aligned on main axis
 - Inner nodes: subdivision planes
 - Leaf nodes: triangles



Example



Example with triangles overlapping the split



Ray Tracing with kd tree

- Goal: find closest hit with scene
 - Traverse tree front to back
 - (starting from root)
- At each node:
 - If leaf: intersect with triangles
 - If inner: traverse deeper

Testing BVH's

```
TestBVH(A,B) {
  if(not overlap(ABV, BBV) return FALSE;
  else if(isLeaf(A)) {
    if(isLeaf(B)) {
      for each triangle pair (Ta,Tb)
        if(overlap(Ta,Tb)) AddIntersectionToList();
    }
    else {
      for each child Cb of B
        TestBVH(A,Cb);
    }
  }
  else {
    for each child Ca of A
      TestBVH(Ca,B)
  }
}
```

Optimization Structures

- All of these optimization structures can be used in either 2D or 3D
- Packing in memory may affect caching and performance

Pair Reduction

- At a minimum, any moving object should have some sort of bounding sphere (or other simple primitive)
- Before a pair of objects is tested in any detail, we can quickly test if their bounding spheres intersect
- When there are lots of moving objects, even this quick bounding sphere test can take too long, as it must be applied N^2 times if there are N objects
- Reducing this N^2 problem is called *pair reduction*
- Pair testing isn't a big issue until $N > 50$ or so...

Appendix

Collision Response

Impact vs. Contact

- In physics simulation, there is usually a distinction between impacts and contacts
- Impacts are instantaneous collisions between objects where an impulse must be generated to prevent the velocities at the impact location from allowing the objects to interpenetrate
- Contacts are persistent and exist over some range of time. In a contact situation, the closing velocities at the contact location should already be 0, so forces are needed to keep the objects from accelerating into each other. With rigid bodies, contacts can include fairly complex situations like stacking, rolling, and sliding

Impact vs. Contact

- Neither impact nor contact is particularly easy to handle correctly
- In the case of particles, it's not so bad, but with rigid bodies, it can be tough
- As we are mainly just concerned with the physics of particles, we will not worry about the more complex issues for now
- Also, we will just focus on handling impacts, as they are generally needed first. Continuous contact will just be handled by allowing particles to impact frame after frame

Impacts

- When two solid objects collide (such as a particle hitting a solid surface), forces are generated at the impact location that prevent the objects from interpenetrating
- These forces act over a very small time and as far as the simulation is concerned, it's easiest to treat it as an instantaneous event
- Therefore, instead of the impact applying a force, we must use an impulse

Impulse

- An impulse can be thought of as the integral of a force over some time range, which results in a finite change in momentum:

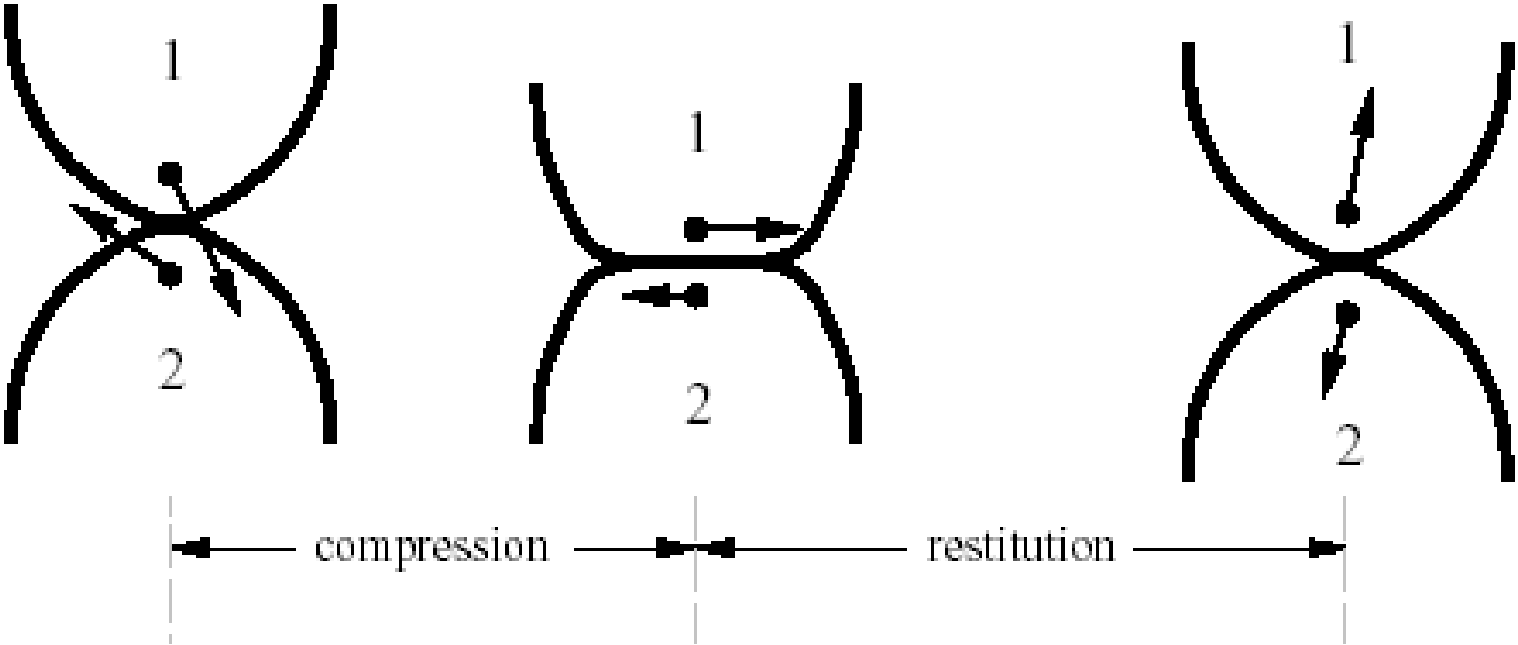
$$j = \int f dt = \Delta p$$

- An impulse behaves a lot like a force, except instead of affecting an object's acceleration, it directly affects the velocity
- Impulses also obey Newton's Third Law, and so objects can exchange equal and opposite impulses
- Also, like forces, we can compute a total impulse as the sum of several individual impulses

Compression & Restitution

- The collision can be thought of as having two phases: compression & restitution
- In the compression phase, the energy of the two objects is changed from kinetic energy of motion into deformation energy in the solids
- If the collision is perfectly *inelastic* ($e=0$), then all of the energy is lost and there will be no relative motion along the collision normal after the collision
- If the collision is perfectly *elastic* ($e=1$), then all of the deformation energy will be turned back into kinetic energy in the restitution phase and the velocity along the normal will be the opposite of what it was before the collision

Compression & Restitution



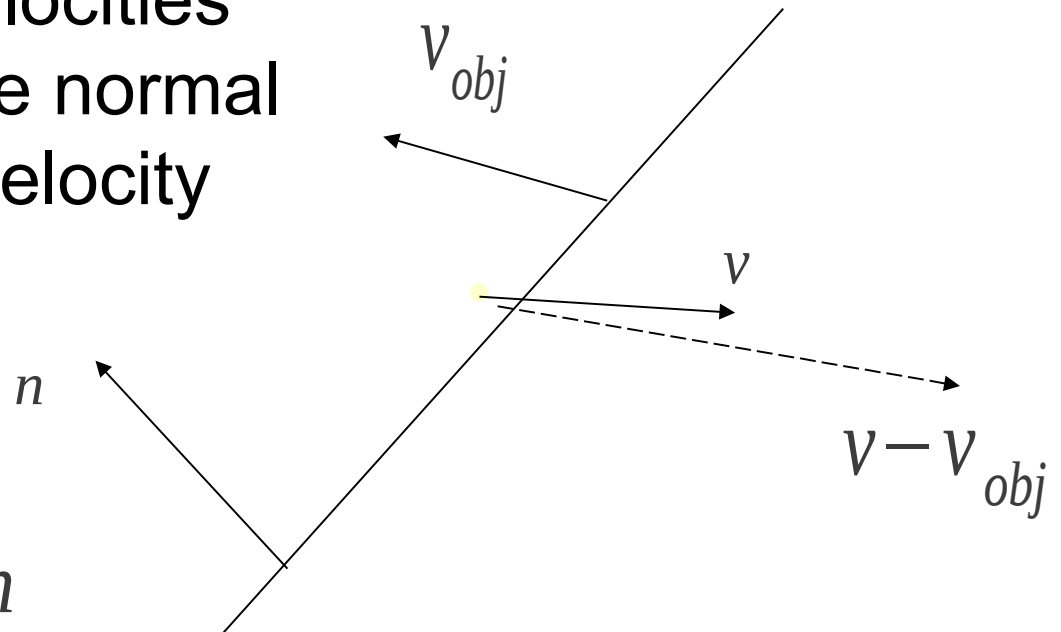
Collisions

- Consider the case of a particle colliding with a heavy object. The object is moving with velocity \mathbf{v}_{obj}
- The particle has a velocity of \mathbf{v} before the collision and collides with the surface with a unit normal \mathbf{n}
- We want to find the collision impulse \mathbf{j} applied to the particle during the collision

Collisions

- We take the difference between the two velocities and dot that with the normal to find the closing velocity

$$v_{close} = (v - v_{obj}) \cdot n$$



Collisions

- Let's first consider a collision with no friction
- The collision impulse will be perpendicular to the collision plane (i.e., along the normal) and will be large enough to stop the particle (at least)

$$j = -(1 + e)mv_{close}n$$

Friction

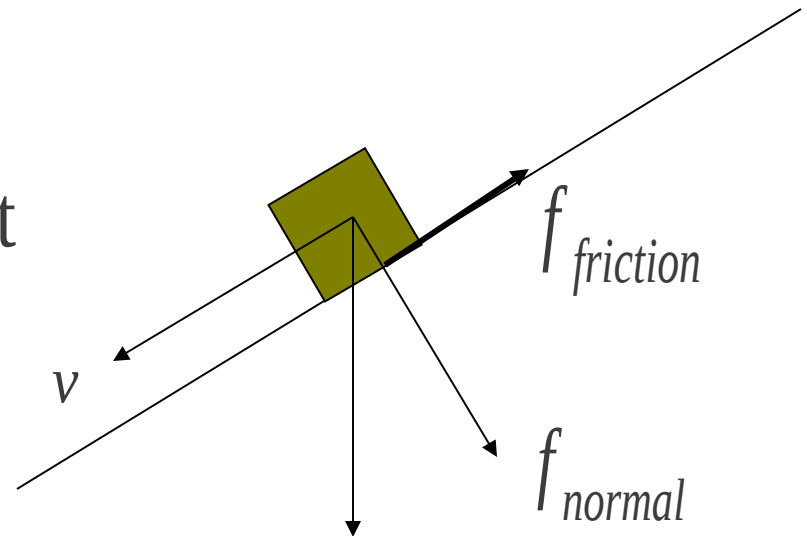
- The Coulomb friction model says:

$$f_{dynamic} = \mu_d |f_{normal}| e$$

$$f_{static} \leq \mu_s |f_{normal}| e$$

μ_d : dynamic friction coefficient

μ_s : static friction coefficient



Friction

- As we are not considering static contact, we will just use a single dynamic friction equation
- For an impact, we can just compute the impulse in the exact same way as we would for dynamic friction
- We can use the magnitude of the elasticity impulse as the normal impulse

$$j_{dynamic} = \mu_d |j_{normal}| e$$

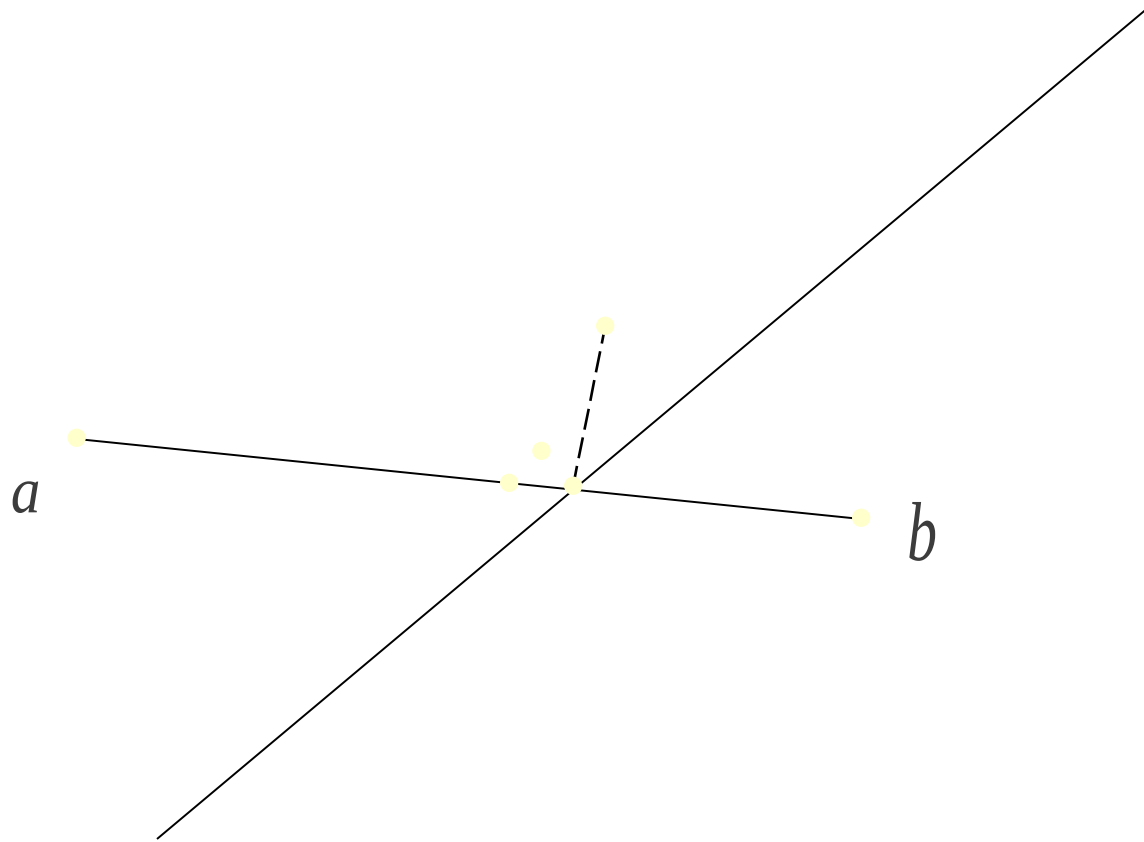
Collision Handling

- For particles and cloth, the following approach works effectively:
 1. Compute forces (springs, aero...)
 2. Integrate motion (Euler step)
 3. Test if particles hit anything
 - 3.1 Compute & apply impulse (adjust velocity)
 - 3.2 Adjust position

Position Adjustment

- Moving the particle to a legal position isn't always easy
- There are different possibilities:
 - Move it to a position just before the collision
 - Put it at the collision point
 - Put it at the collision point plus some offset along the normal
 - Compute where it would have gone if it bounced
- Computing the bounced position is really the best, but may involve more computation and in order to do it right, it may require further collision testing...

Position Adjustment



Bouncing

- Computing the bounced position is the best approach, as it is consistent with the rest of the physics model
- We need to determine when exactly the collision happened (we can just assume that the particle traveled at a constant velocity within the frame)
- We then compute the impulse and adjust the velocity
- Then, we move the particle forward by the amount of time remaining within the frame
- Ideally, we should then check collisions on this new path
- A particle getting stuck in a narrow crack might bounce several times, so we should put a cap on the maximum number of bounces allowed, then just stop the particle at some point if it exceeds the limit